

3 (Sem-1) MAT M 1 (O)

2019

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×10=10

(a) Does the set of all integers form a group with respect to addition of integers?

(b) What is the degree of the following permutation?

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

(2)

- (c) Write Lagrange's theorem (Group theory).
- (d) Write generators of the multiplicative cyclic group

$$\{1, -1, i, -i\}$$

- (e) Find the argument of the complex number $-1 + i\sqrt{3}$.

- (f) Express $\cosh y$ in the power of e^y and e^{-y} .

- (g) Use Gregory's series, find the value of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \infty$$

- (h) Define symmetric matrix.
- (i) What is the normal form of a matrix?
- (j) What is the echelon form of the matrix $A = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$?

(3)

2. Answer the following questions : $2 \times 5 = 10$

- (a) Give with an example that union of two subgroups of a group is not necessarily a subgroup of the group.

- (b) Express the following matrix as a sum of symmetric and skew-symmetric matrices :

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$$

- (c) If

$$A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$$

then show that B is the inverse of A .

- (d) If A is an $n \times n$ non-singular matrix, then prove that

$$|\text{adj } A| = |A|^{n-1}$$

- (e) Write $-i$ in the form

$$r(\cos \theta + i \sin \theta)$$

(4)

3. Answer the following questions : $5 \times 2 = 10$

(a) Prove that every subgroup of a cyclic group is cyclic.

(b) Express $\log_e(x+iy)$ in the form $A+iB$, where $x, y \in R$.

Or

Prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$ if $i^{\alpha+\beta} = \alpha + i\beta$.

4. Answer any two questions of the following : $5 \times 2 = 10$

(a) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then find the value of $\Sigma \alpha^3$ in terms of p, q and r .

(5)

(b) If the roots of the equation

$$x^3 - ax^2 + bx - c = 0$$

be in harmonic progression, then show that the mean root is $\frac{3c}{b}$.

(c) Apply Descartes' rule of signs to find the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$.

5. Answer any one part : 10

(a) Let A be a non-empty set and let R be an equivalence relation in A . Let a and b be arbitrary elements in A . Then prove that—

(i) $[a] = [b]$ iff $(a, b) \in R$;

(ii) either $[a] = [b]$ or $[a] \cap [b] = \phi$.

(b) State and prove fundamental theorem on equivalence relation.

6. Answer any one part : 10

(a) (i) If H is any subgroup of G and $h \in H$, then prove that $Hh = H = hH$.

(ii) If a, b are any two elements of a group G and H any subgroup of G , then prove that $Ha = Hb \Leftrightarrow ab^{-1} \in H$.

(b) If H is a subgroup of G , then prove that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right cosets of H in G .

7. Answer any one part : 10

(a) Separate into real and imaginary parts of $(\alpha + i\beta)^{x+iy}$.

(b) If $\sin(\alpha + i\beta) = x + iy$, then prove that—

(i) $x^2 \operatorname{cosec}^2 \alpha - y^2 \sec^2 \alpha = 1;$

(ii) $x^2 \operatorname{sech}^2 \beta + y^2 \operatorname{cosech}^2 \beta = 1.$

8. Answer any one part : 10

(a) For what values of η , the equations

$$x + y + z = 1$$

$$x + 2y + 4z = \eta$$

$$x + 4y + 10z = \eta^2$$

have a solution? Solve them completely in each case.

(b) Prove that every square matrix A can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices.
