Ex) Obtain $\sqrt{12}$, to five places of decimals by Newton-Raphson method.

Sol^n: Let us take

$$x = \sqrt{12}$$

$$\Rightarrow x^2 - 12 = 0$$

Let $f(x) = x^2 - 12$

From this it is obvious that $x^2 - 12$ becomes negative for $x = 3$, whereas it is positive for $x = 3.5$. Hence the root of $x^2 - 12 = 0$ lies in the interval $(3, 3.5)$.

Again, $f'(x) = 2x$

Now, by Newton-Raphson method, we have,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad [\text{taking } x_0 = 3.5]$$

$$= 3.5 - \frac{(3.5)^2 - 12}{2 \times 3.5}$$

$$= 3.46428$$

[If x_0 is not given, then we take two numbers a and b such that $f(a)\,f(b) < 0$, then the root α lies in betn a and b. Now, $x_0 = a$, if $f(a)$ is nearer to zero than $f(b)$ or $x_0 = b$, if $f(b)$ is nearer to zero than $f(a)$]

Again,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 3.46428 - \frac{(3.46428)^2 - 12}{2 \times 3.46428}$$

$$= 3.46410$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 3.46410 - \frac{(3.46410)^2 - 12}{2 \times 3.46410}$$

$$= 3.46410$$

Hence upto five places of decimal, we see

$$\sqrt{12} = 3.46410$$

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Ex) Find the cube root of 10 by using Newton-Raphson formula.

Sol^{n}:: Let us take

$$x = 10^{1/3}$$

$$\Rightarrow x^3 - 10 = 0$$

Let $f(x) = x^3 - 10$

From this it is obvious that $x^3 - 10$ becomes negative for $x = 2$, whereas it is positive for $x = 3$, Hence the root of $x^3 - 10 = 0$ lies in the interval $(2, 3)$

Again, $f'(x) = 3x^2$

Now, by Newton-Raphson method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad [\text{taking } x_0 = 2]$$

$$= 2 - \frac{f(2)}{f'(2)} = 2 - \frac{-2}{12} = 2.1667$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.1667 - \frac{f(2.1667)}{f'(2.1667)}$$

$$= 2.1667 - \frac{.17177}{14.08377}$$

$$= 2.15450$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.15450 - \frac{f(2.15450)}{f'(2.15450)}$$

$$= 2.15443$$

Hence the required cube root is 2.154 //

Ex) Using Newton-Raphson method, find an iterative formula to compute the reciprocal of a natural number.

Sol^n: Let $x = \frac{1}{N}$

$\Rightarrow N - \frac{1}{x} = 0$

also, let $f(x) = N - \frac{1}{x} = 0$

and $f'(x) = \frac{1}{x^2}$

Now, from Newton-Raphson formula, we have,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{N - \frac{1}{x_n}}{\frac{1}{x_n^2}}$$

$$= x_n - N x_n^2 + x_n$$

$$\Rightarrow x_{n+1} = x_n(2 - N x_n)$$

which is the required iterative formula to find $\frac{1}{N}$.

*Ex) Find the real root of the eqn $x^2 - 5x + 2 = 0$ correct to four places of decimal by using Newton-Raphson method.

Sol^n: Let $f(x) = x^2 - 5x + 2$

$\therefore f'(x) = 2x - 5$

Now, $f(0) = 2 > 0$ and $f(1) = -2 < 0$.

So a real root lies betn 0 and 1.

But for quick convergence we take $x_0 = 0.5$.

Now, from Newton-Raphson formula, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ for $n = 0$

$= 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.4375$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= .4375 - \frac{f(.4375)}{f'(.4375)}$$

$$= .4384$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= .4384 - \frac{f(.4384)}{f'(.4384)}$$

$$= .4384$$

Hence the reqd. value of the root is .4384 //