Introduction to Normal distribution:

The normal distribution was first discovered by De-Moivre in 1733 which is sometimes also called as Gaussian distribution.The Normal distribution is also a limiting form of the Binomial distribution under the following conditions:

- *n*, the number of trials is indenfinitely large i.e., $n\to\infty$.
- neither *p* nor *q* is very small.

Mathematical form:

A continuous random variable X is said to follow normal distribution with mean μ and varince σ^2 if its probability density function is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{-2\sigma^2}}, \quad -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

Here, μ and σ^2 are the two parameters of normal distribution and we can write $X \sim N(\mu, \sigma^2)$, i.e., X follows normal distribution with mean μ and variance σ^2.

Standard normal variate:

If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma}$ is called a standard normal variate. The mean and variance of a standard normal variate are respectively 0 and 1 i.e., Z ~ N (0, 1).

$$E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}[E(X) - \mu] = \frac{1}{\sigma}[\mu - \mu] = 0$$

and
$$V(Z) = V\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}[V(X) - 0] = \frac{1}{\sigma^2}[\sigma^2] = 1$$

The p.d.f. of standard normal distribution is given by,

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{z^2}{-2}}, \quad -\infty < z < \infty$$

and we can write Z ~ N (0, 1).

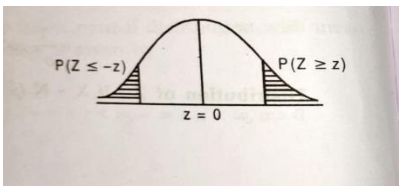
Distribution function of Z:

The distribution function of Z is

$$P(Z \le z) = \emptyset(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{t^2}{-2}} dt$$

The distribution function of standard normal variate Z satisfies the following results:

- $\emptyset(-z) = P(Z \le -z) = P(Z \ge z) = 1 - P(Z \le z)$, by symmetry.



- For any a < b

$$P(a < X < b) = P\left\{\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right\}$$
$$= P\left\{\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right\}$$
$$= P\left\{Z < \frac{b-\mu}{\sigma}\right\} - P\left\{Z < \frac{a-\mu}{\sigma}\right\}$$
$$= \emptyset\left\{\frac{b-\mu}{\sigma}\right\} - \emptyset\left\{\frac{a-\mu}{\sigma}\right\}$$

Properties of normal distribution:

- The normal curve is bell-shaped and symmetrical about the line $x=\mu$.
- The distribution is unimodel.
- Mean = Median = Mode = μ.

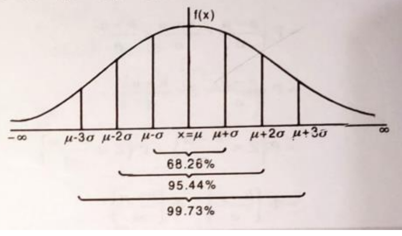
- The two quartiles Q_1 and Q_3 are equidistant from the mean μ and

$$Q_1 = \mu - 0.6745\sigma$$

$$Q_3 = \mu + 0.6745\sigma$$

- The points of inflexion of the curve are at $x = \mu + \sigma$ and $x = \mu - \sigma$.
- Q.D : M.D : S.D : : 10 : 12 : 15
- M.D.$=\frac{4}{5}\sigma$, Q.D.$=\frac{2}{3}\sigma$
- All odd order central moments are zero.
- Area property of normal distribution:

2. $P(\mu - \sigma < X < \mu + \sigma) = 0.6826$
$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$
$P(\mu - 3\sigma < X < \mu - 3\sigma) = 0.9973$



Calculation of probabilities-use of tables:

The areas under the standard normal curve for different values of z are available in statistical tables. The ordinates and areas of the standard normal distribution are available in the statistical tables for different z values from -∞ to z for different positive values of z. Here ordinate is denoted by $\varphi(z)$ and area is denoted by $\emptyset(z).\emptyset(z) = P(Z \le z)$. The values of $\emptyset(-z)$ for negative values of z can be obtained from the symmetry of the p.d.f.

$$\emptyset(-z) = P(Z \le -z) = P(Z \ge z) = 1 - P(Z \le z) = 1 - \emptyset(z)$$

The reason is that the area under the curve to the left of -z is the same as the area under the curve to the right of z. Moreover, the total area of the curve is 1.

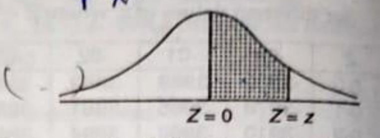
TABLE 9·2. AREAS UNDER NORMAL CURVE

Normal probability curve is given by :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, -\infty < x < \infty$$

and standard normal probability curve is given by :

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), -\infty < z < \infty$$



where $Z = \frac{X - E(X)}{\sigma_X} \sim N(0, 1)$

The following table gives the shaded area in the diagram, *viz.*, $P(0 < Z < z)$ for different values of z.

TABLE OF AREAS

↓Z→	0	1	2	3	4	5	6	7	8	9
·0	·0000	·0040	·0080	·0120	·0160	·0199	·0239	·0279	·0319	·0359
·1	·0398	·0438	·0478	·0517	·0557	·0596	·0636	·0675	·0714	·0759
·2	·0793	·0832	·0871	·0910	·0948	·0987	·1026	·1064	·1103	·1141
·3	·1179	·1217	·1255	·1293	·1331	·1368	·1406	·1443	·1480	·1517
·4	·1554	·1591	·1628	·1664	·1700	·1736	·1772	·1808	·1844	·1879
·5	·1915	·1950	·1985	·2019	·2054	·2088	·2123	·2157	·2190	·2224
·6	·2257	·2291	·2324	·2357	·2389	·2422	·2454	·2486	·2517	·2549
·7	·2580	·2611	·2642	·2673	·2703	·2734	·2764	·2794	·2823	·2852
·8	·2881	·2910	·2939	·2967	·2995	·3023	·3051	·3078	·3106	·3133
·9	·3159	·3186	·3212	·3238	·3264	·3289	·3315	·3340	·3365	·3389
1·0	·3413	·3438	·3461	·3485	·3508	·3531	·3554	·3577	·3599	·3621
1·1	·3643	·3655	·3686	·3708	·3729	·3749	·3770	·3790	·3810	·3830
1·2	·3849	·3869	·3888	·3907	·3925	·3944	·3962	·3980	·3997	·4015
1·3	·4032	·4049	·4066	·4082	·4099	·4115	·4131	·4147	·4162	·4177
1·4	·4192	·4207	·4222	·4236	·4251	·4265	·4279	·4292	·4306	·4319
1·5	·4332	·4345	·4357	·4370	·4382	·4394	·4406	·4418	·4429	·4441
1·6	·4452	·4463	·4474	·4484	·4495	·4505	·4515	·4525	·4535	·4545
1·7	·4554	·4564	·4573	·4582	·4591	·4599	·4608	·4616	·4625	·4633
1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
2·1	·4821	·4826	·4830	·4834	·4838	·4842	·4846	·4850	·4854	·4857
2·2	·4861	·4864	·4868	·4871	·4875	·4678	·4881	·4884	·4887	·4890
2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936
2·5	·4938	·4940	·4941	·4943	·4945	·4946	·4948	·4959	·4951	·4952
2·6	·4953	·4955	·4956	·4957	·4959	·4960	·4961	·4962	·4963	·4964
2·7	·4965	·4966	·4967	·4968	·4969	·4970	·4971	·4972	·4973	·4974
2·8	·4974	·4975	·4976	·4977	·4977	·4978	·4979	·4979	·4980	·4981
2·9	·4981	·4982	·4982	·4983	·4984	·4984	·4985	·4985	·4986	·4986
3·0	·4987	·4987	·4987	·4988	·4988	·4989	·4989	·4989	·4990	·4990
3·1	·4990	·4991	·4991	·4991	·4992	·4992	·4992	·4992	·4993	·4993
3·2	·4993	·4993	·4994	·4994	·4994	·4994	·4994	·4995	·4995	·4995
3·3	·4995	·4995	·4995	·4996	·4996	·4996	·4996	·4996	·4996	·4997
3·4	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4998
3·5	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998
3·6	·4998	·4998	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·7	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·9	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000

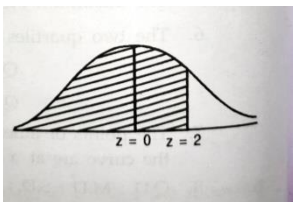
- If X is a normal variate with mean 5 and standard deviation 4. Find

  (i) P(X < 13)   (ii) P(X > 1)   (iii) P (4 < X < 9)

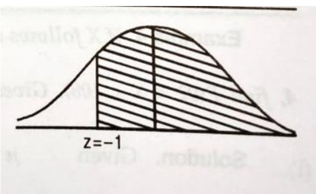
  - Given, $\mu = 5$, $\sigma = 4$

    $\therefore \quad Z = \frac{X-\mu}{\sigma} = \frac{X-5}{4}$

(i) $P(X < 13) = P(\frac{X-5}{4} < \frac{13-5}{4}) = P(Z < 2) = \emptyset(2) = 0.5 + 0.4772 = 0.9772$

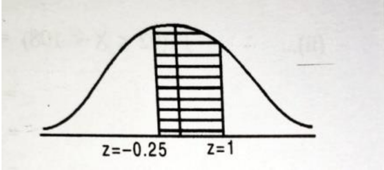


(ii) $P(X > 1) = P\left(\frac{X-5}{4} > \frac{1-5}{4}\right) = P(Z > -1) = P(Z < 1) = \emptyset(1) = 0.5 + 0.3413 = 0.8413$



(iii) $P(4 < X < 9) = P\left(\frac{4-5}{4} < \frac{X-5}{4} < \frac{9-5}{4}\right) = P(-0.25 < Z < 1) = \emptyset(1) - \emptyset(-0.25)$

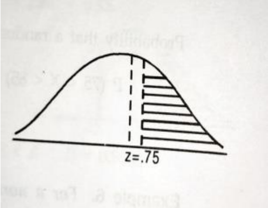
$= (0.5 + 0.3413) - [1 - \emptyset(0.25)] = 0.8413 - 1 + (0.5 + 0987) = 0.44$



- If X ~ N (15, 16), find the probabilty that X is larger than 18.
  - Given μ=15, σ^2= 16, which gives σ=4

Now, $P(X > 18) = P\left(\frac{X-15}{4} > \frac{18-15}{4}\right) = P(Z > 0.75) = 1 - P(Z < 0.75) = 1 - \emptyset(0.75)$

$= 1 - \{0.5 + 0.2734\} = 0.2266$



- The marks obtained in a Statistics examination are assumed to have a normal distribution with mean 75 and variance 25. Find the probability of a rondomly selected student obtaining marks:

  (i) Between 65 and 75, (ii) Below 70.

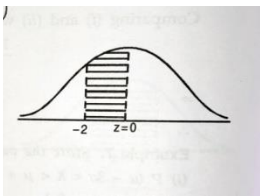
It is given that, $P\left(\frac{X-75}{5} < 2\right) = 0.9772$, $P\left(\frac{X-75}{5} < 1\right) = 0.8413$.

  - Given, μ=75, σ^2= 25, which gives σ=5

  We have to find the probability *P (65<X<75)*, *P(X<70)*

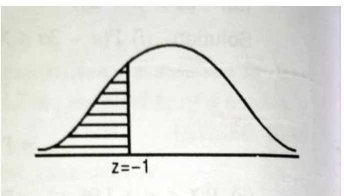
(i) $P(65 < X < 75) = P\left(\frac{65-75}{5} < \frac{X-75}{5} < \frac{75-75}{5}\right) = P(-2 < Z < 0) = \emptyset(0) - \emptyset(-2)$

$= 0.5 - [1 - \emptyset(2)] = 0.5 - 1 + 0.9772 = 0.4772$



(ii) $P(X < 70) = P\left(\frac{X-75}{5} < \frac{70-75}{5}\right) = P(Z < -1) = P(Z > 1) = 1 - P(Z < 1)$

$= 1 - \emptyset(1) = 1 - 0.8413 = 0.1587$



- The marks obtained in a Statistics examination are assumed to have a normal distribution with mean 80 and variance 25. Find the probability of a randomly selected student obtaining marks betwen 75 and 85.
  - Given, $\mu=80$, $\sigma^2= 25$, which gives $\sigma=5$

  We have to find the probability $P\ (75<X<85)$.

*****TRY IT YOURSELF*****