

Normal form of a Matrix (or Canonical form):

By performing a series of elementary (row or column) transformations, any non-zero matrix A can be reduced to one of the following four forms, called the normal forms of A .

(i) $[I_r]$, (ii) $[I_r \ 0]$, (iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$, (iv) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

The number r , so obtained is called the rank of the matrix A i.e. $\rho(A) = r$.

Procedure to obtain normal form:

- (i) Interchange rows or columns to obtain a non-zero element in first row and first column of the given matrix.
- (ii) If this non-zero element is not 1, then make it 1.
- (iii) Obtain zeros in the remainder of 1st row and 1st column.
- (iv) Repeat the above three steps starting with element in the 2nd row and 2nd column.
- (v) Continue the procedure down the main diagonal either until the end of the diagonal is reached OR all the elements which are zero.

Ex) Find the rank of the matrix by normal form,

where

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

Soln:-

Given

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + 2R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$C_2 \rightarrow C_2 - 2C_1$$

$$C_3 \rightarrow C_3 - 3C_1$$

$$C_4 \rightarrow C_4 + C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$R_4 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 3 & 3 & -3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 2R_2$$

$$R_4 \rightarrow R_4 - 3R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 \rightarrow C_3 - C_2$$

$$C_4 \rightarrow C_4 + C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \text{Rank of } A = 2$$