* Newton-Raphson Method:

Let x_0 be an approximate value of a root of the eqn $f(x)=0$ and let x_0+h be the exact value of the corresponding root, where h is very small quantity.

Then $f(x_0+h)=0$ ---(1), since x_0+h is the root of the eqn $f(x)=0$.

Expanding (1) by Taylor's thm, we get,

$$f(x_0+h) = f(x_0) + h f'(x_0) + \frac{h^2}{\lfloor 2} f''(x_0) + \cdots = 0$$

Since h is very small, neglecting second and higher order terms and taking the first approximation, we have,

$$f(x_0) + h f'(x_0) = 0$$

$$\Rightarrow h = -\frac{f(x_0)}{f'(x_0)}, \quad \text{Provided } f'(x_0) \neq 0$$

$\therefore \quad x_1 = x_0 + h$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \text{-----} (2)$$

Relation (2) gives the improved value of the root over the previous one. Now substituting x_1 for x_0 and x_2 for x_1 in (2), we get

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \text{-----} (3)$$

In general, we can get an approximation or the iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{-----} (4)$$

From this formula, we can calculate successive better values of the root. Formula (4) is known as Newton - Raphson Method.

From the above result, it is obvious that this method is applicable only when h is very small, i.e., when $f'(x)$ is large, in other words when the graph of the f^n near the root is nearly verticle.

Ex) Find a real root of the eqn $3x - \cos x - 1 = 0$ correct to four significant figures by using Newton-Raphson method.

Solⁿ: Let $f(x) = 3x - \cos x - 1$

$\therefore f'(x) = 3 + \sin x$

Here, $f(0) = -2$ and $f(1) = 1.4597$

$\therefore$ a real root lies betⁿ 0 and 1.

We take $x_0 = 1$

Now, by Newton-Raphson formula, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad [\text{for } n=0]$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \frac{1.4597}{3.84147}$$

$$= .620015$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= .620015 - \frac{f(.620015)}{f'(.620015)} \qquad [\text{for } n=1]$$

$$= .620015 - \frac{.04618}{3.58105}$$

$$= .60712$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad [\text{for } n=2]$$

$$= .60712 - \frac{f(.60712)}{f'(.60712)}$$

$$= .60712 - \frac{.00006552}{3.57050}$$

$$= .60710$$

Thus the positive real root of the given eqn is 0.6071 correct to four significant figures. //