A special transformation:

We have already made use of transformations in reducing both homogeneous and Bernoulli equations to more tractable types. Another type of equation that can be reduced to a more basic type by means of a suitable transformation is an equation of the form

$$(a_1 x + b_1 y + c_1)\, dx + (a_2 x + b_2 y + c_2)\, dy = 0$$

We state the following theorem concerning this equation.

※ Consider the equation

$$(a_1 x + b_1 y + c_1)\, dx + (a_2 x + b_2 y + c_2)\, dy = 0 \quad \cdots (1)$$

where a_1, b_1, c_1, a_2, b_2 and c_2 are constants.

Case I: If $\frac{a_2}{a_1} \neq \frac{b_2}{b_1}$, then the transformation

$$x = X + h$$
$$y = Y + k$$

where (h, k) is the solution of the system

$$a_1 h + b_1 k + c_1 = 0$$
$$a_2 h + b_2 k + c_2 = 0$$

reduces equation (1) to the homogeneous equation

$$(a_1 X + b_1 Y)\, dX + (a_2 X + b_2 Y)\, dY = 0$$

in the variables X and Y.

Case II: If $\frac{a_2}{a_1} = \frac{b_2}{b_1} = k$, then the transformation $z = a_1 x + b_1 y$ reduces the equation (1) to a separable equation in the variables x and z.