Ex) Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

Soln: Dividing by $\cos^2 y$,

$$\sec^2 y \frac{dy}{dx} + \frac{x \sin 2y}{\cos^2 y} = x^3$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} + \frac{2x \cos y \sin y}{\cos^2 y} = x^3$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} + 2x \tan y = x^3 \quad \text{---(1)}$$

Put $\tan y = v$

$$\Rightarrow \frac{dv}{dx} = \sec^2 y \frac{dy}{dx}$$

$\therefore$ (1) $\Rightarrow$ $\frac{dv}{dx} + 2xv = x^3$

Here I.F $= e^{\int 2x\,dx} = e^{x^2}$

Hence the reqd. soln is

$$v \cdot e^{x^2} = \int x^3 \cdot e^{x^2} dx + c$$

$$= \frac{1}{2} \int t e^t \, dt + c \qquad \left[\text{Put } x^2 = t \Rightarrow 2x\,dx = dt\right]$$

$$= \frac{1}{2} \left[t e^t - \int 1 \cdot e^t \, dt \right] + c$$

$$= \frac{1}{2} (te^t - e^t) + c$$

$$\Rightarrow \tan y \, e^{x^2} = \frac{1}{2} (x^2 - 1) e^{x^2} + c$$

$$\Rightarrow \tan y = \frac{1}{2} (x^2 - 1) + c e^{-x^2}$$

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Ex) Solve: $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

Sol^n:

Given eq^n is

$$\frac{dy}{dx} = e^{2x} \cdot e^{-y} - e^x$$

$$\Rightarrow \frac{dy}{dx} + e^x = e^{2x} \cdot e^{-y}$$

$$\Rightarrow e^y \frac{dy}{dx} + e^x \cdot e^y = e^{2x} \quad \cdots (1)$$

Put $e^y = v$

$$\Rightarrow \frac{dv}{dx} = e^y \frac{dy}{dx}$$

$\therefore (1) \Rightarrow \frac{dv}{dx} + e^x \cdot v = e^{2x}$

Here I.F. $= e^{\int e^x dx} = e^{e^x}$

Hence the reqd. Sol^n is

$$v \cdot e^{e^x} = \int e^{2x} \cdot e^{e^x} dx + c$$

$$= \int t e^t dt + c \qquad \left[\text{Put } e^x = t \Rightarrow e^x dx = dt\right]$$

$$= t e^t - \int 1 \cdot e^t dt + c$$

$$= t e^t - e^t + c$$

$$= e^t (t-1) + c$$

$$\Rightarrow e^y e^{e^x} = e^{e^x}(e^x - 1) + c$$

$$\Rightarrow e^y = (e^x - 1) + c e^{-e^x}$$

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