* The secant method:

The secant method is very close to the regula-falsi method except of the necessity of the condition $f(x_0)\, f(x_1) < 0$ for the interval $[x_0, x_1]$ in which the root lies. Here also the graph of the f^n $y = f(x)$ is approximated by a secant line (chord) in the neighbourhood of the root.

If $[x_0, x_1]$ be the initial interval in which the root α of the eq^n $f(x) = 0$ lies, then the eq^n of the chord joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ is

$$y - f(x_1) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}(x - x_1)$$

which intersects the x-axis at $(x_2, 0)$, where

$$x_2 = x_1 - \frac{f(x_1)}{f(x_1) - f(x_0)}(x_1 - x_0)$$

Repeating the process, the $(n+1)$th iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(x_{n-1})}(x_n - x_{n-1})$$

Ex) A real root of the eq^n

$$f(x) = x^3 - 5x + 1 = 0$$

lies in the interval $(0, 1)$. Perform four iterations of the secant method and the Regular - Falsi method to obtain this root.

Sol^n:

We have,

$$x_0 = 0, \quad x_1 = 1$$

$$f(x_0) = 1, \quad f(x_1) = -3$$

Secant method:

$$x_2 = x_1 - \frac{(x_1 - x_0)}{\{f(x_1) - f(x_0)\}} f(x_1)$$

$$= 1 - \frac{1}{-4}(-3) = 0.25$$

$$f(x_2) = -.234375$$

$$x_3 = x_2 - \frac{(x_2 - x_1)}{\{f(x_2) - f(x_1)\}} f(x_2)$$

$$= .25 - \frac{(.25 - 1)}{(-.234375 + 3)}(-.234375)$$

$$= .186441$$

$$f(x_3) = .074276$$

$$x_4 = x_3 - \frac{(x_3 - x_2)}{\{f(x_3) - f(x_2)\}} f(x_3)$$

$$= .201736$$

$$f(x_4) = -0.000470$$

$$x_5 = x_4 - \frac{(x_4 - x_3)}{\{f(x_4) - f(x_3)\}} f(x_4)$$

$$= .201736 - \frac{(.201736 - .186441)}{(-.000470 - .074276)}(-.000470)$$

$$= .201640$$

Hence the required real root of the given eq^n is $.201640$

Regula - Falsi Method:

$$x_2 = x_1 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] f(x_1)$$

$$= 0.25$$

$$f(x_2) = f(.25) = -0.234375$$

since, $f(x_0)\, f(x_2) < 0$

$\therefore$ the root lies betn (x_0, x_2) i.e. $(0, 0.25)$

$$\therefore\ x_3 = x_2 - \left[\frac{x_2 - x_0}{f(x_2) - f(x_0)}\right] f(x_2)$$

$$= 0.25 - \left[\frac{0.25 - 0}{-.234375 - 1}\right](-.234375)$$

$$= .202532$$

$$f(x_3) = f(.202532) = -.004352$$

since, $f(x_0)\, f(x_3) < 0$

$\therefore$ the root lies betn (x_0, x_3) i.e. $(0, .202532)$

$$\therefore\ x_4 = x_3 - \left\{\frac{x_3 - x_0}{f(x_3) - f(x_0)}\right\} f(x_3)$$

$$= .202532 - \left\{\frac{.202532 - 0}{-.004352 - 1}\right\}(-.004352)$$

$$= .201654$$

since, $f(x_0)\, f(x_4) < 0$

$\therefore$ the root lies betn (x_0, x_4) i.e. $(0, .201654)$

$$\therefore\ x_5 = x_4 - \left[\frac{x_4 - x_0}{f(x_4) - f(x_0)}\right] f(x_4)$$

$$= .201640$$

Hence the required real root of the given eqn is

$\therefore$ .201640 //