PRACTICALS ON POISSON DISTRIBUTION

Problem: After correcting 50 pages of the proof of a book, the proof reader finds that there are, on the average, 2 errors per 5 pages. How many pages would one expect to find with 0, 1, 2, 3 and 4 errors, in 1,000 pages of the first print of the book? (Given that $e^{-0.4} = 0.6703$)

Aim: - Fitting of Poisson distribution for given value of λ.

Theory: The p. m. f. of Poisson distribution is given by,

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x = 0, 1, 2, \ldots, \infty$$

Using reccurence formula for the probabilities of Poisson distribution, we have

$$p(x+1) = \frac{\lambda}{(x+1)}.p(x)$$

and the expected frequencies are given by the $f(x) = N.\ p(x)$; N= Total frequency

Calculation: We are given, the mean number of errors per page, $\lambda = \frac{2}{5} = 0.4$, $N = 1000$.

Hence $p(0) = P(X = 0) = \frac{e^{-0.4}(0.4)^{0}}{0!} = e^{-0.4} = 0.6703$

Similarly we get the probabilities $p(1), p(2), \ldots, p(4)$ by using reccurence formula as follows:

$$p(1) = p(x+1)_0 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_0 = \frac{0.4}{(0+1)}.p(0) = 0.4 \times 0.6703 = 0.2681$$

$$p(2) = p(x+1)_1 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_1 = \frac{0.4}{(1+1)}.p(1) = \frac{0.4}{2} \times 0.6703 = 0.0536$$

$$p(3) = p(x+1)_2 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_2 = \frac{0.4}{(2+1)}.p(2) = \frac{0.4}{3} \times 0.0536 = 0.0071$$

$$p(4) = p(x+1)_3 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_3 = \frac{0.4}{(3+1)}.p(3) = \frac{0.4}{4} \times 0.0071 = 0.00071$$

And the expected frequencies are calculated as:

$$f(0) = N \times p(0) = 1000 \times 0.6703 = 670.3 \cong 670$$

Now, we prepare the following table:

Table 1

x	$\frac{\lambda}{(x+1)}$	$p(x)$	$f(x) = N.\ p(x)$
0	0.4	0.6703	$f(0) = N \times p(0) = 1000 \times 0.6703 = 670.3 \cong 670$
1	0.2	0.2681	$f(1) = N \times p(1) = 1000 \times 0.2681 = 268.1 \cong 268$
2	0.4/3	0.0536	$f(2) = N \times p(2) = 1000 \times 0.0536 = 53.6 \cong 54$
3	0.1	0.0071	$f(3) = N \times p(3) = 1000 \times 0.0071 = 7.1 \cong 7$
4	-	0.00071	$f(4) = N \times p(4) = 1000 \times 0.00071 = 0.71 \cong 1$

Conclusion: The Poisson distribution is fitted.

Problem: Fit a Poisson distribution to the following data which gives the number of doddens in a sample of clover seeds.

No. of doddens (x)	0	1	2	3	4	5	6	7	8
Observed frequencies (f)	56	156	132	92	37	22	4	0	1

Aim: - Fitting of Poisson distribution by computing its mean value λ.

Theory: The p. m. f. of Poisson distribution is given by,

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x = 0, 1, 2, \ldots, \infty$$

Using reccurence formula for the probabilities of Poisson distribution, we have

$$p(x+1) = \frac{\lambda}{(x+1)}.p(x)$$

and the expected frequencies are given by the $f(x) = N.\ p(x)$; N= Total frequency

Calculation: Here, the mean value is not given. So first of all, we have to find the mean value from the given data as follows:

No. of doddens (x)	0	1	2	3	4	5	6	7	8	Total
Observed frequencies (f)	56	156	132	92	37	22	4	0	1	N=500
fx	0	156	264	276	148	110	24	0	8	$\sum fx = 986$

$$E(X) = \bar{X} = \lambda = \frac{1}{N}\sum fx = \frac{986}{500} = 1.972$$

Hence $p(0) = P(X = 0) = \frac{e^{-1.972}(1.972)^0}{0!} = e^{-1.972} = 0.1392$

Similarly we get the probabilities $p(1), p(2), \ldots, p(8)$ by using reccurence formula as follows:

$$p(1) = p(x+1)_0 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_0 = \frac{1.972}{(0+1)}.p(0) = 1.972 \times 0.1392 = 0.2745$$

$$p(2) = p(x+1)_1 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_1 = \frac{1.972}{(1+1)}.p(1) = \frac{1.972}{2} \times 0.2745 = ---$$

$$p(3) = p(x+1)_2 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_2 = \frac{1.972}{(2+1)}.p(2) = \frac{1.972}{3} \times \ldots = ---$$

$$p(4) = p(x+1)_3 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_3 = \frac{1.972}{(3+1)}.p(3) = \frac{0.1972}{4} \times \ldots = ---$$

$$p(5) = p(x+1)_4 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_4 = \frac{1.972}{(4+1)}.p(3) = \frac{1.972}{5} \times \ldots = ---$$

$$p(6) = p(x+1)_5 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_5 = \frac{1.972}{(5+1)}.p(3) = \frac{1.972}{6} \times \ldots = ---$$

$$p(7) = p(x+1)_6 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_6 = \frac{1.972}{(6+1)}.p(3) = \frac{1.972}{7} \times \ldots = ---$$

$$p(8) = p(x+1)_7 = \left\{\frac{\lambda}{(x+1)}.p(x)\right\}_7 = \frac{1.972}{(7+1)}.p(3) = \frac{1.972}{8} \times \ldots = ---$$

And the expected frequencies are calculated as:

$$f(0) = N \times p(0) = 500 \times 0.1392 = 69.6 \cong 70$$

Now, we prepare the following table:

Table 1

x	$\frac{\lambda}{(x+1)}$	$p(x)$	$f(x) = N.\,p(x)$
0	1.972	0.1392	$f(0) = N \times p(0) = 500 \times 0.1392 = 69.6 \cong 70$
1	0.986	0.2745	$f(1) = N \times p(1) = 500 \times 0.2745 = 137.25 \cong 137$
2	0.657	---	$f(2) = N \times p(2) = ---$
3	0.493	---	$f(3) = N \times p(3) = ---$
4	0.394	---	$f(4) = N \times p(4) = ---$
5	0.329	---	$f(5) = N \times p(5) = ---$
6	0.282	---	$f(6) = N \times p(6) = ---$
7	0.247	---	$f(7) = N \times p(7) = ---$
8	0.219	---	$f(8) = N \times p(8) = ---$

Conclusion: The Poisson distribution is fitted.

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