Equations reducible to linear form:

An equation of the form

$$f'(y)\frac{dy}{dx} + Pf(y) = Q \quad \cdots (1)$$

where P and Q are constants or functions of x alone can be reduced to linear form as follows:

Putting $f(y) = v$ so that $f'(y)\frac{dy}{dx} = \frac{dv}{dx}$,

$\therefore (1) \Rightarrow$ $$\frac{dv}{dx} + Pv = Q \quad \cdots (2)$$

which is linear in v and x and its solution can be obtained by using $I.F = e^{\int P\,dx}$ and solution is $v.\, e^{\int P\,dx} = \int Q\, e^{\int P\,dx}\,dx + c$. Finally, replace v by $f(y)$ to get solution in terms of x and y alone.

Similarly,

An equation of the form

$$f'(x)\frac{dx}{dy} + P_1 f(x) = Q_1 \quad \cdots (3)$$

where P_1 and Q_1 are constants or functions of y alone can be reduced to linear form by putting $f(x) = v$

$$\Rightarrow f'(x)\frac{dx}{dy} = \frac{dv}{dy}$$

$\therefore (3) \Rightarrow$ $$\frac{dv}{dy} + P_1 v = Q_1$$

which is linear in variables v and y and its solution can be obtained by using

$I.F. = e^{\int P_1 dy}$ and solution is

$$v . e^{\int P_1 dy} = \int Q_1 e^{\int P_1 dy} dy + C$$

Replacing v by $f(x)$, we obtain the required sol^n of (3).