Ex) Solve: $(x+2y^3)\frac{dy}{dx} = y$

Sol^n:- Here it is possible to put the equation in form

$$\frac{dx}{dy} + P_1 x = Q_1$$

Thus we have,

$$\frac{dx}{dy} = \frac{x+2y^3}{y}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 2y^2 \quad \text{----- (1)}$$

Here $\text{I.F.} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}$

Hence the required solution is

$$x.\frac{1}{y} = \int 2y^2.\frac{1}{y}\,dy + c$$

$$= 2.\frac{y^2}{2} + c$$

$$\Rightarrow \frac{x}{y} = y^2 + c,$$ where c is an arbitrary constant.

Ex) Solve: $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Sol^n: Here $\frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\therefore \text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

Hence required sol^n is

$$x \cdot e^{\tan^{-1}y} = \int e^{\tan^{-1}y} \cdot \frac{\tan^{-1}y}{1+y^2}\, dy + c$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int e^{t} \cdot t \, dt + c$$

$$\left[\begin{array}{l} \text{Put } \tan^{-1}y = t \\ \therefore \frac{dy}{1+y^2} = dt \end{array}\right.$$

$$= t \int e^t dt - \int \left(\frac{d}{dt} t \int e^t dt\right) dt + c$$

$$= t e^t - \int e^t dt + c$$

$$= t e^t - e^t + c$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \tan^{-1}y \, e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

$$= e^{\tan^{-1}y}(\tan^{-1}y - 1) + c \quad //$$

Ex) Solve:

$$(\sec x \tan x \tan y - e^x)\, dx + \sec x \sec^2 y \, dy = 0$$

Sol^n: We have,

$$\sec x \tan x \tan y - e^x + \sec x \sec^2 y \frac{dy}{dx} = 0$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} + \tan x \tan y = e^x \cos x \quad \text{---(1)}$$

Put $\tan y = z \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore$ (1) $\Rightarrow$

$$\frac{dz}{dx} + z \tan x = e^x \cos x$$

Here $\quad I.F. = e^{\int \tan x \, dx}$

$$= e^{\log \sec x} = \sec x$$

Hence the required sol^n is

$$z \sec x = \int e^x \cos x \sec x \, dx + c$$

$$\Rightarrow z \sec x = \int e^x \, dx + c$$

$$= e^x + c$$

$$\Rightarrow \tan y \sec x = e^x + c \quad //$$

Ex) Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2$, if $y=1$ when $x=1$.

Sol^m:-

Here $I.F = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ Reqd. sol^n is

$$y \cdot x = \int x^2 \cdot x \, dx + c$$

$$\Rightarrow y x = \frac{x^4}{4} + c \quad \text{---} (1)$$

Given $y = 1$ when $x = 1$

$\therefore (1) \Rightarrow \quad 1 = \frac{1}{4} + c \quad \Rightarrow c = \frac{3}{4}$

$\therefore$ Reqd. sol^n is

$$y x = \frac{x^4}{4} + \frac{3}{4}$$

$$\Rightarrow 4xy = x^4 + 3 \quad //$$