6.4. RECURRENCE RELATION FOR THE PROBABILITIES OF POISSON DISTRIBUTION

$$p(x) = \frac{e^{-\lambda}\,\lambda^{x}}{x!}$$

$$\therefore \quad p(x+1) = \frac{e^{-\lambda}\,\lambda^{x+1}}{(x+1)!}$$

$$= \frac{e^{-\lambda}\,\lambda^{x}}{x!}\,\frac{\lambda}{x+1} = \frac{\lambda}{x+1}\,p(x)$$

Example 1. *X follows the Posson distributon and P(X) = 1) = P(X = 2) ; find P(X = 4).* (AHSEC 2000)

Solution. If $X \sim p(\lambda)$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} ; x = 0, 1, 2, \ldots$$

Given, $P(X = 1) = P(X = 2)$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow \quad \lambda = \frac{\lambda^2}{2}$$

$$\Rightarrow \quad \lambda = 2$$

$$\therefore \quad P(X = 4) = \frac{e^{-2}2^4}{4!}$$

$$= \frac{16}{24} \times .135$$

$$= 0.09.$$

Example 2. *X follows Poisson distribution and P(X = 0) = P(X = 1) = a. Show that* $a = \frac{1}{e}$. (AHSEC 1996)

Solution. Given $P(X = 0) = P(X = 1)$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}\lambda^1}{1!}$$

$$\Rightarrow \quad \lambda = 1$$

Also, $P(X = 0) = a$

$$\Rightarrow \quad \frac{e^{-1}1^0}{0!} = a$$

$$\Rightarrow \quad e^{-1} = a$$

$$\Rightarrow \quad a = \frac{1}{e}.$$

Example 3. *If X has a Poisson distribution and* $P(X = 0) = \frac{1}{2}$, *what is E(X) ?* (AHSEC 1997)

Solution. Given $P(X = 0) = \frac{1}{2}$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^0}{0!} = \frac{1}{2}$$

$$\Rightarrow \quad e^{-\lambda} = \frac{1}{2}$$

$$\Rightarrow \quad -\lambda = \log_e \left(\frac{1}{2}\right)$$

$$\Rightarrow \quad -\lambda = \log_e 1 - \log_e 2$$

$$\Rightarrow \quad -\lambda = 0 - .693$$

$$\Rightarrow \quad -\lambda = -.693$$

$$\Rightarrow \quad E(X) = \lambda = 0.693$$

Example 4. *A random variable X is such that* **P(X = 1) = 2. P(X = 2).** *Find (i) P(X = 0), (ii) P(X ≥ 0).*

Solution. Given, P(X = 1) = 2 P(X = 2)

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^1}{0!} = \frac{e^{-\lambda}\lambda^2}{2!}.2$$

$$\Rightarrow \quad \lambda = \lambda^2$$

$$\Rightarrow \quad \lambda = 1$$

(*i*) $$P(X = 0) = \frac{e^{-1}1^0}{0!}$$

$$= e^{-1} = .368$$

(*ii*) $$P(X \geq 0) = 1$$

Example 5. *From a population consisting of defective and non-defective items, 100*