Rank of a Matrix:

A number r is said to be the rank of a matrix A, if it possesses the following two properties:

(i) The determinant of atleast one minor of order r is non-zero.

(ii) The determinant of every minor of A of order higher than r is zero.

Rank of a matrix A is denoted by $\rho(A)$.

Properties of Rank:

(i) The rank of every non-singular matrix of order n is n.

(ii) The rank of a square matrix A of order n can be less than n if and only if A is singular i.e. $|A| = 0$

(iii) The rank of a zero matrix is taken as zero.

(iv) If I_n is a unit matrix of order n, then its rank is equal to n.

(v) Elementary transformations do not alter the rank of a matrix.

Examples:

(a) Let $A = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a unit matrix of order 3. We have $|A| = 1$. Therefore A is a non-singular matrix. Hence rank of $A = 3$.

(b) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Since A is a null matrix, therefore rank of $A = 0$

(c) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

We have $|A| = 1(6-8) - 2(4-0) + 3(4-0)$

$= 2 \neq 0$

Thus A is a non-singular matrix.

Therefore rank $A = 3$.

Ex) find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

<u>Sol:-</u> Here $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{vmatrix} = 1 - 4 + 3 = 0$

Its 3rd order minor is zero, so we calculate the second order minors.

One second order minor is

$$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0.$$

∴ Rank of the given matrix is 2.