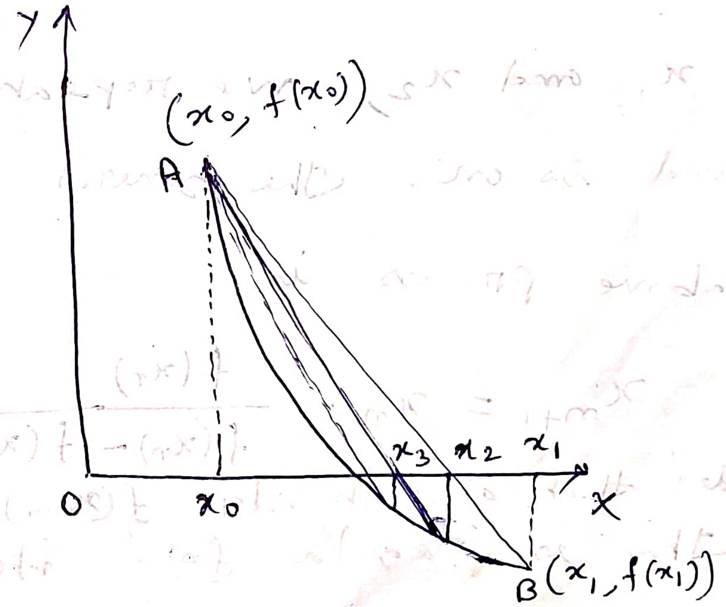
* The method of Regula Falsi or False position:



The method of regula falsi or false position is also sometimes referred to as the method of linear interpolation and it is the oldest method of computing real roots of an eqn $f(x)=0$. To find a real root α of $f(x)=0$, we first choose a sufficiently small interval $[x_0, x_1]$ in which the root α lies.

Then $f(x_0)$ and $f(x_1)$ must be of opposite signs so that $f(x_0)\, f(x_1) < 0$ and the graph of $f(x)$ must cross the x-axis betn $x = x_0$ and $x = x_1$. Since the interval $[x_0, x_1]$ is sufficiently small, the portion of the curve betn $A[x_0, f(x_0)]$ and $B[x_1, f(x_1)]$ can be well approximated by a secant line (straight line) and so the intersection of the secant AB with the x-axis gives an approximate value x_2 (say) of the root.

Using the slope formula, the eqn of the secant line AB is

$$y - f(x_1) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}(x - x_1)$$

putting $y=0$, $x = x_2$, we have

$$x_2 = x_1 - \frac{f(x_1)}{f(x_1) - f(x_0)}(x_1 - x_0)$$

We repeat the process to obtain x_3 and so on. The general formula based on the above process is

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(x_{n-1})}(x_n - x_{n-1}),$$

Provided that at each step $f(x_{n-1})\, f(x_n) < 0$.

This is regula falsi iteration formula.

Ex) Find a real root of the eqn $x^3 - 2x - 5 = 0$ by the method of false position, correct to three decimal places.

Solⁿ: Let $f(x) = x^3 - 2x - 5$, then we have,

$f(2) = -1$ and $f(3) = 16$

$\therefore$ a root lies betⁿ 2 and 3.

Let $x_0 = 2$, $x_1 = 3$

$\therefore$ $f(x_0) = -1$ and $f(x_1) = 16$

$$\therefore \quad x_2 = x_1 - \frac{(x_1 - x_0)}{(f(x_1) - f(x_0))} f(x_1) = 3 - \frac{(3-2)}{(16+1)} \times 16$$

$$= 2.0588$$

$f(x_2) = f(2.0588) = -.3911$

since $f(2.0588) \cdot f(3) < 0$

$\therefore$ the root lies betn $(2.0588, 3)$

$$\therefore x_3 = 3 - \frac{(3 - 2.0588)}{16 + .3911} \times 16$$

$$= 2.0813$$

$f(x_3) = f(2.0813) = -.1468$

Since $f(2.0813) f(3) < 0$

$\therefore$ the root lies betn $(2.0813, 3)$

$$\therefore x_4 = 3 - \frac{(3 - 2.0813)}{16 + .1468} \times 16$$

$$= 2.0896$$

$f(x_4) = f(2.0896) = -.0551$

Since $f(2.0896) f(3) < 0$

$\therefore$ the root lies betn $(2.0896, 3)$

$$\therefore x_5 = 3 - \frac{(3 - 2.0896)}{16 + .0551} \times 16$$

$$= 2.0927$$

$f(x_5) = f(2.0927) = -.0206$

since $f(2.0927) f(3) < 0$

$\therefore$ the root lies betn $(2.0927, 3)$

$$\therefore x_6 = 3 - \frac{(3 - 2.0927)}{16 + .0206} \times 16$$

$$= 2.0938 \quad \text{etc.}$$

Hence the root is 2.094 correct to 3 decimal places.