Ex) Perform five iterations of the bisection method to obtain the smallest positive root of the eqn $x^3 - 5x + 1 = 0$.

Soln :- Since $f(0) > 0$ and $f(1) < 0$, the smallest positive root lies in the interval $(0, 1)$.

Taking $a_0 = 0$, $b_0 = 1$, we set,

$$m_1 = \frac{1}{2}(a_0 + b_0) = 0.5$$

$$f(m_1) = f(0.5) = -1.375$$

and $f(0.5)\, f(0) < 0$

Thus, the root lies in the interval $(0, 0.5)$.

Taking $a_1 = 0$, $b_1 = 0.5$, we get

$$m_2 = \frac{1}{2}(a_1 + b_1) = 0.25$$

$$f(m_2) = f(0.25) = -0.234375$$

and $f(0) f(0.25) < 0$

Thus the root lies in the interval $(0, 0.25)$.

The sequence of intervals is given ~~in the~~ below:

n	a_n	b_n	$m_{n+1} = \left(\frac{a_n + b_n}{2}\right)$	$f(m_{n+1})$
0	0	1	0.5	-1.375
1	0	0.5	.25	-0.234375
2	0	.25	0.125	.37695
3	0.125	0.25	0.1875	0.06909
4	0.1875	0.25	.21875	-0.08328

Hence the root lies in $(0.1875, .21875)$.

The approximate root is taken as the mid point of this interval, i.e., 0.203125 //

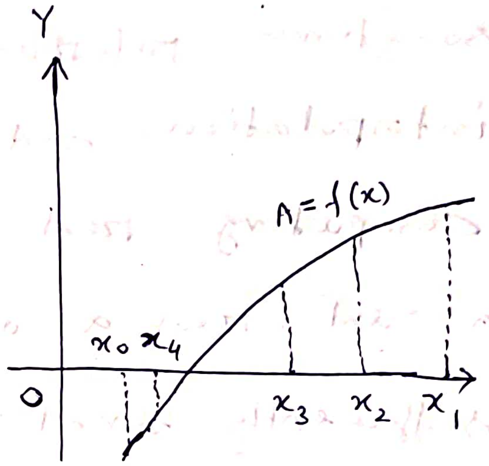
Ex) Find an approximate root of the eqn $x^3 - 4x - 9 = 0$ using the bisection method four times.

B.Sc

Ex) Explain the bisection method with suitable diagram;

Ans:-

This method is based on the th^m ~~that~~ which states that "If a f^n $f(x)$ is continuous in the closed interval $[x_0, x_1]$ and $f(x_0)$ and $f(x_1)$ are of opposite signs [i.e, if $f(x_0)f(x_1) < 0$] then there exists at least one real root of $f(x)=0$ between x_0 and x_1



Let $f(x_0)$ and $f(x_1)$ be of opposite signs i.e. $f(x_0)f(x_1) < 0$. Then the desired root is approximately

$$x_2 = \frac{x_0 + x_1}{2}$$

Now $x_3 = \frac{x_0 + x_2}{2}$ Provided that $f(x_0)f(x_2) < 0$ i.e, if it lies bet^n x_0 and x_2.

and $x_3 = \frac{x_1 + x_2}{2}$ Provided $f(x_1)f(x_2) < 0$.

Further $x_4 = \frac{x_0 + x_3}{2}$ Provided $f(x_0)f(x_3) < 0$ and so on.

Thus, in each iteration we either find the root with desired accuracy or we narrow the range to half the previous interval.