Sometimes a differential equation which is not exact may become so, on multiplying a suitable function known as the integrating factor.

1) If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function of x alone, say $f(x)$, then $IF = e^{\int f(x)\,dx}$

Example: $(2x\log x - xy)\,dy + 2y\,dx = 0$

Here $M = 2y$ and $N = 2x\log x - xy$

Hence $\dfrac{\partial M}{\partial y} = 2$ and $\dfrac{\partial N}{\partial x} = 2\log x + 2 - y$
$= 2(1+\log x) - y$

Here $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{-2\log x + y}{2x\log x - xy} = -\dfrac{1}{x} = f(x)$

$\therefore\ IF = e^{\int f(x)\,dx} = e^{\int -\frac{1}{x}\,dx} = e^{-\log x} = e^{\log \frac{1}{x}}$
$= \dfrac{1}{x}$

on multiplying the given eqn by $\frac{1}{x}$, we get
$(2\log x - y)\,dy + \dfrac{2}{x}y\,dx = 0$

2) If $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$ is a function of y alone, say $f(y)$, then $IF = e^{\int f(y)\,dy}$

Example: $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

Here $M = y^4 + 2y$ and $N = xy^3 + 2y^4 - 4x$

$\therefore\ \dfrac{\partial M}{\partial y} = 4y^3 + 2$ and $\dfrac{\partial N}{\partial x} = y^3 - 4$

$$\therefore \quad \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3 - 4 - (4y^3 + 2)}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3+2)}$$

$$= \frac{-3(y^3+2)}{y(y^3+2)} = -\frac{3}{y} = f(y)$$

$$\therefore \quad IF = e^{\int f(y)\,dy} = e^{\int -\frac{3}{y}\,dy} = e^{-3\log y}$$

$$= e^{\log y^{-3}} = y^{-3} = \frac{1}{y^3}$$

On multiplying the given eqn by $\frac{1}{y^3}$, we get the exact differential equation as

$$\left(y + \frac{2}{y^2}\right) dx + \left(x + 2y - \frac{4x}{y^3}\right) dy = 0$$