* Interpolation with equal Intervals:

Interpolation means insertion or filling up intermediate terms of a series. It is the technique of obtaining the value of a function for any intermediate values of the independent variable when the values of the function corresponding to a number of the values of the variable are given.

Interpolation with equal intervals when one or more of the values of $f(x)$ corresponding to the equidistant values of x are missing, then the following method may be used for finding the missing value:

we have,

$$\Delta^n f(a) = (E-1)^n f(a)$$

$$= \left[E^n - {}^nC_1 E^{n-1} + {}^nC_2 E^{n-2} - \cdots + (-1)^n \right] f(a)$$

$$= E^n f(a) - {}^nC_1 E^{n-1} f(a) + {}^nC_2 E^{n-2} f(a) - \cdots + (-1)^n f(a)$$

$$= f(a+nh) - {}^nC_1 f[a+(n-1)h] + {}^nC_2 f[a+(n-2)h] - \cdots + (-1)^n f(a)$$

Assume that $f(x)$ is a polynomial of degree $(n-1)$ in x so that $\Delta^n f(x) = 0$ for all x.

$\therefore f(a+nh) - {}^nc_1 f[a+(n-1)h] + \cdots + (-1)^n f(a) = 0$

This eqn enables us to find out the missing values of $y = f(x)$ when only n values of $f(x)$ are known.

Ex) Estimate the missing term in the following table:

x:	1	2	3	4	5	6	7
y:	2	4	8	?	32	64	128

Explain why the result difference from 2^4.

Sol^n:- Here 6 values of $f(x)$ are given.

$\therefore \Delta^6 f(x) = 0$

$\Rightarrow (E-1)^6 f(x) = 0$

$\Rightarrow (E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) f(x) = 0$

for $x = 1$,

$(E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) f(1) = 0$ [$\because h = 1$]

$\Rightarrow f(6+1) - 6f(5+1) + 15f(4+1) - 20f(3+1) + 15f(2+1) - 6f(1+1) + f(1) = 0$

$\Rightarrow f(7) - 6f(6) + 15f(5) - 20f(4) + 15f(3) - 6f(2) + f(1) = 0$

$\Rightarrow 128 - 6 \times 64 + 15 \times 32 - 20 \times f(4) + 15 \times 8 - 6 \times 4 + 2 = 0$

$\Rightarrow f(4) = \frac{322}{20} = 16.1$

~~$f(x) = 2^x$ in a~~

This value differs with the value 2^4 because in obtaining the resulting value, we have assumed $f(x)$ as a polynomial function but originally it is the exponential function $f(x) = 2^x$

[Power variable in exp^{fn}]

Ex) obtain the missing term in the following table:

x:	0	1	2	3	4
$f(x)$:	1	3	9	?	81

Ans:- $f(3) = 31$

Ex) obtain the missing terms in the following table:

x:	2·0	2·1	2·2	2·3	2·4	2·5	2·6
y_x:	·135	?	·111	·100	?	·082	·074

Sol^n:- Here five values of y_x are given.

$\therefore \quad \Delta^5 y_x = 0 \quad \forall x$

$\Rightarrow (E-1)^5 y_x = 0$

$\Rightarrow (E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_x = 0 \quad \cdots (1)$

Here $h = 0.1$.

Putting $x = 2.0$ in (1), we get,

$(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_{2.0} = 0$

$\Rightarrow y_{2.5} - 5y_{2.4} + 10y_{2.3} - 10y_{2.2} + 5y_{2.1} - y_{2.0} = 0$

$\Rightarrow ·082 - 5x\, y_{2.4} + 10x·100 - 10x·111 + 5x\, y_{2.1} - ·135 = 0$

$\Rightarrow 5y_{2.1} - 5y_{2.4} - ·163 = 0 \quad \cdots (2)$

Again, putting $x = 2.1$ in (1), we get,

$y_{2.6} - 5y_{2.5} + 10y_{2.4} - 10y_{2.3} + 5y_{2.2} - y_{2.1} = 0$

$\Rightarrow ·074 - 5x·082 + 10y_{2.4} - 10x·100 + 5x·111 - y_{2.1} = 0$

$\Rightarrow -y_{2.1} + 10y_{2.4} - ·781 = 0$

$\Rightarrow -5y_{2.1} + 50y_{2.4} - 3·905 = 0 \quad \cdots (3)$

(2) + (3) $\Rightarrow \quad 45y_{2.4} = 4·068$

$\Rightarrow y_{2.4} = 0·0904$

Putting $y_{2\cdot4} = 0.0904$ in (2), we get

$$5y_{2\cdot1} - 5 \times 0.0904 - 0.163 = 0$$

$$\Rightarrow 5y_{2\cdot1} = 0.615$$

$$\Rightarrow y_{2\cdot1} = 0.123$$ //