Discrete Probability Distributions

Binomial Distn :- The Binomial distn was derived by James Bernoulli in 1700. First of all, we have to study about Bernoulli trials, Bernoulli variate and Bernoulli distn which are described below:-

Bernoulli Distn:- Let X be a discrete random variable which takes only two values 0, with prob^y q and 1, with prob^y p, i.e.

$$P(X=0) = q = \text{prob}^y \text{ of failure}$$

$$P(X=1) = p = \text{prob}^y \text{ of success.}$$

Then the distn of X is said to be Bernoulli distn whose p.m.f. (prob^y mass fuction) is given by,

$$P(X=x) = p^x q^{1-x} \quad ; \quad x = 0, 1$$
$$= 0, \quad \text{otherwise}$$

Here the variate X is called Bernoulli variate as it takes only two values (i.e., 0, 1).

Again, a random experiment whose outcomes are of two types, success (S) and Failure (F) occuring with prob^y p, q respectively is called a Bernoulli trial.

(+) **Binomial distn and its derivation** :- Let us consider n independent Bernoulli trials, where n is finite. The prob^y of success (p) in any trial is constant and $q = 1-p$ is the prob^y of failure. Suppose, we are interested in the prob^y of obtaining x successes in n trials.

Then the n trials results in a sequence of n outcomes each of which may be either success (S) or failure (F). One such sequence is :-

$$\underbrace{S, S, \ldots, S}_{x \text{ times}} \quad \underbrace{F, F, \ldots, F}_{(n-x) \text{ times}}$$

Since, the n trials are independent of one another, then by multiplicative law, we have

$$P(S, S, S, \ldots, S, F, F, \ldots, F) = P(S).P(S)\ldots P(S).P(F).P(F).\ldots P(F)$$

$$= \underset{(x \text{ times})}{p.p.\ldots, p} \quad \underset{(n-x) \text{ times}}{q.q.\ldots q} \qquad \left(\because \begin{array}{l} P(S) = p \\ P(F) = q \end{array}\right)$$

$$= p^x q^{n-x}$$

But, the no. of sequences involving x successes in n trials will be ^nC_x. Hence we have, the required prob^n will be :- $^nC_x p^x q^{n-x}$, which is the p.m.f. of Binomial dist^n.

Thus, in other words, we can say that :-

If X is a discrete r.v. with p.m.f.

$$P(X=x) = {}^nC_x p^x q^{n-x}; \quad x = 0, 1, 2, \ldots, n$$
$$= 0, \quad \text{otherwise}$$

and $p + q = 1$,

then X is said to follow Binomial dist^n with parameter

n, p and we can write $X \sim B(n, p)$.

(+) <u>Physical conditions for Binomial distn:-</u>

(1) Each trial results in two exhaustive & mutually disjoint outcomes, termed as success and failure.

(2) The no. of trials, n is finite.

(3) The prob^{y} of success p is constant for each trial

(4) The trials are independent of each other.