Definition: Zero - Divisors

A nonzero element a in a Commutative ring R is called a zero divisor if there is a nonzero element b in R such that $ab = 0$.

Definition: Integral Domain

A commutative ring with a unity is said to be an integral domain if it has no zero-divisors.

Thus, in an integral domain, a product is 0 only when one of the factor is 0; that is $ab=0$ only when $a=0$ or $b=0$.

Ex. The ring of integers is an integral domain.

Pf: The set of integers is a commutative ring. [Already proved] with unity.

Let, for $a, b \in Z$, $ab = 0$.

Then either $a = 0$ or $b = 0$.

$\therefore$ Z has no zero-divisors. Hence Z is an integral domain.

Ex. The ring of Gaussian integers

$Z[i] = \{a+bi \mid a, b \in Z\}$ is an integral domain.

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Ex: The ring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is an integral domain.

Ex. The ring $\mathbb{Z}_p$ of integers modulo p is an integral domain.

Ex: The ring $\mathbb{Z}_n$ of integers modulo n is not an integral domain when n is not prime.

Ex. The ring $M_2(\mathbb{Z})$ of 2×2 matrices over the integers is not an integral domain

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$\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$

for $a, b \in \mathbb{Z}_n$

$a +_n b$ ~~called~~ addition modulo n is the remainder when $a+b$ is divided by n.

$a \times_n b$ ~~called~~ multiplication modulo n is the remainder when $a \times b$ is divided by n

for eg. Take $a = 3$, $b = 4$, $n = 6$

Then $a +_6 b = 3 +_6 4 = 1$

$a \times_n b = 3 \times_6 4 = 0$.

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$Z_n = \{0, 1, 2, \ldots, n-1\}$

$+_n$	0	1	2	. . .	$n-1$
0	0	1	2	. . .	$n-1$
1	1	2	3	. . .	0
2	2	3	4	. . .	1
⋮	⋮				
$n-1$	$n-1$	0	1	. . .	$n-2$

$2+n-1$
$n+1$
$=1$

$2n-2$

$n+n-2$

$\times_n$	0	1	2	. . .	$n-1$
0	0	0	0	. . .	0
1	0	1	2	. . .	$n-1$
2	0	2	4	. . .	$n-2$
3	0	3	6	. . .	$n-3$
⋮	⋮			. . .	1
$n-1$	0	$n-1$	$n-2$		

$2(n-1)$
$= 2n-2$
$= n+n-2$

$3(n-1)$
$= 3n-3$
$= 2n+n-3$

$(n-1)(n-1)$
$= n^2-2n+1$

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