Control Charts for Attributes

The variable control charts can be used only for measurable quality characteristics and these charts are sometimes become impracticable in certain situations. Thus, as an alternative of variable control charts, one can use the control charts for attributes which can be applied for the quality characteristics:

- Which can be observed only as attributes by classifying them as defective/ non-defective items.
- Which are actually observed as attributes even though they could be measured as variables.

There are two control charts for attributes-

- Control chart for fraction defective (p-chart)
- Control chart for the number of defects per unit (c-chart)

Control chart for fraction defective or p – chart:

If d is the number of defectives in a sample of size n, then the sample proportion defective is $p=\frac{d}{n}$. Hence d is a binomial variate with parameters n and P.

Hence, $E(d) = nP$, $V(d) = nPQ$, where $Q=1-P$

which gives, $E(p) = E(\frac{d}{n})= \frac{1}{n}E(d)=P$, $V(p) = V(\frac{d}{n})= \frac{1}{n^2}V(d)= \frac{PQ}{n}$,

Thus, the 3-σ control limits for p-chart are given by,

$$E(p) \pm 3 \text{ S. E. } (p) = P \pm 3\sqrt{\frac{PQ}{n}} = P \pm A\sqrt{PQ}$$

Where $A=\frac{3}{\sqrt{n}}$ has been tabulated for different values of n.

Case I: (When standards are given)

If P' is a given or known value of p, then

$$\text{UCL} = P' + A\sqrt{PQ}$$

$$\text{LCL} = P' - A\sqrt{PQ}$$

$$\text{CL} = P'$$

Case II: (When standards are not given)

If d_i is the number of defectives and p_i is the fraction defective for the i^{th} sample $(i=1, 2 \ldots, k)$ of size n_i, then the population proportion p is estimated by the sample statistic $\bar{p}$ which is given by

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

$\bar{p}$ is an unbiased estimate of p. So

$$\text{UCL} = \bar{p} + A\sqrt{\bar{p}(1-\bar{p})}$$

$$\text{LCL} = \bar{p} - A\sqrt{\bar{p}}(1-\bar{p})$$

$$\text{CL} = \bar{p}$$

p-chart for fixed sample size:

If the sample size remains constant for each sample, i.e., if $n_{1,}n_{2,} \ldots, n_k = n$ (say). Then an estimate of the population proportion p is given by,

$$\hat{p} = \bar{p} = \frac{\sum np_i}{\sum n} = \frac{n\sum p_i}{nk} = \frac{1}{k}\sum p_i$$

Control chart for number of defects or c-chart:

An item is considered to be defective if it fails to conform to specifications in any characteristics. Each characteristic that does not meet the specification is a defect. An item is defective if it contains at least one defect. For instance, a radio set may have many defects, viz., its bulbs may be out of order, circuit may be wrong etc. But from the angle of the number of defective pieces, it is only one defective piece, whereas from the point of view of number of defects, they are many. The probability of a defect in a manufactured piece is small, hence the variable will follow Poisson distribution. In this case, the quality characteristic, which is to be plotted on the control chart is the average no. of defects per piece. This average no. of defects is denoted by c and hence the control chart for number of defects is known as the c-chart. Since c is a Poisson variate with parameter λ, we get

$$E(c) = \lambda, \qquad V(c) = \lambda$$

Thus 3-σ control limits for c-chart are:

$$\text{UCL} = E(c) + 3\sqrt{V(c)} = \lambda + 3\sqrt{\lambda}$$

$$\text{LCL} = E(c) - 3\sqrt{V(c)} = \lambda - 3\sqrt{\lambda}$$

$$\text{CL} = \lambda$$

<u>Case I:</u> (When standards are given)

If λ' is a given or known value of λ, then

$$\text{UCL} = \lambda' + 3\sqrt{\lambda'}$$

$$\text{LCL} = \lambda' - 3\sqrt{\lambda'}$$

$$\text{CL} = \lambda'$$

<u>Case II:</u> (When standards are not given)

If the value of λ is not known, it is estimated by the mean number of defects per unit. Thus, if c_i is the no. of defects observed on the i^{th} $(i=1, 2,\dots, k)$inspected unit, then an estimate of λ is given by,

$$\hat{\lambda} = \bar{c} = \frac{\sum c_i}{k}$$

$\bar{c}$ is unbiased estimate of λ. Hence, the control limits are:

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

$$\text{CL} = \bar{c}$$

Since c cannot be negative, if LCL given by the above formula comes out to be negative, it is regarded as zero.