* Successive Over Relaxation (SOR) Method :

This method is a generalization of the Gauss-Seidel method.

General formula:

$$x_i^{k+1} = (1-w)x_i^k + \frac{w}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right]$$

Note that when $w = 1$, the SOR method reduces to the Gauss-Seidel method. If $w > 1$, then the method is called an over relaxation method and if $w < 1$, then it is called an under relaxation method.

Here $0 < w < 2$. w is called relaxation parameter.

Ex) Solve the following eqn by SOR method upto three decimal places with $w = 1.25$.

$$27x + 6y - z = 85$$
$$6x + 15y - 2z = 72$$
$$x + y + 54z = 110$$

Soln:- We set up the iteration as

$$x^{k+1} = (1-w)x^k + \frac{w}{27}(85 - 6y^k + z^k)$$
$$y^{k+1} = (1-w)y^k + \frac{w}{15}(72 - 6x^{k+1} + 2z^k)$$
$$z^{k+1} = (1-w)z^k + \frac{w}{54}(110 - x^{k+1} - y^{k+1})$$

For $w = 1.25$,

$$x^{k+1} = -0.25x^k + 0.046(85 - 6y^k + z^k)$$
$$y^{k+1} = -0.25y^k + 0.083(72 - 6x^{k+1} + 2z^k)$$
$$z^{k+1} = -0.25z^k + 0.023(110 - x^{k+1} - y^{k+1})$$

Taking initial values $x = y = z = 0$.

For $K = 0$, (First Iteration)

$$x^{(1)} = -0.25\, x^{(0)} + 0.046\,(85 - 6\,y^{(0)} + z^{(0)})$$

$$= 0 + 0.046\,(85 - 0 + 0)$$

$$= 3.910$$

$$y^{(1)} = -0.25\, y^{(0)} + 0.083\,(72 - 6\cdot x^{(1)} + 2\,z^{(0)})$$

$$= 0 + 0.083\,(72 - 6\times 3.910 + 2\times 0)$$

$$= 4.029$$

$$z^{(1)} = -0.25\, z^{(0)} + 0.023\,(110 - x^{(1)} - y^{(1)})$$

$$= 0 + 0.023\,(110 - 3.910 - 4.029)$$

$$= 2.347$$

For $K = 1$, (2nd Iteration)

$$x^{(2)} = (1-w)\,x^{(1)} + \frac{w}{27}\,(85 - 6\,y^{(1)} + z^{(1)})$$

$$= (1 - 1.25)\times 3.910 + \frac{1.25}{27}\,(85 - 6\times 4.029 + 2.347)$$

$$= -0.978 + 0.046\times(63.173)$$

$$= 1.928$$

$$y^{(2)} = (1-w)\,y^{(1)} + \frac{w}{15}\,(72 - 6\,x^{(2)} + z^{(1)})$$

$$= (1 - 1.25)\times 4.029 + \frac{1.25}{15}\,(72 - 6\times 1.928 + 2.347)$$

$$= -1.007 + 0.083\times(62.779)$$

$$= 4.204$$

$$z^{(2)} = (1-w)\, z^{(1)} + \frac{w}{54}\left(110 - x^{(2)} - y^{(2)}\right)$$

$$= (1-1{\cdot}25) \times 2{\cdot}347 + \frac{1{\cdot}25}{54}\left(110 - 1{\cdot}928 - 4{\cdot}204\right)$$

$$= -0{\cdot}587 + 0{\cdot}023 \times (103{\cdot}868)$$

$$= 1{\cdot}802$$

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