Note:

The operators Δ and E are not commutative with respect to the functions of x i.e. (variables)

if $u_x = f(x).g(x)$, then

$\Delta u_x \neq f(x).\Delta g(x)$

and $E u_x \neq f(x).E g(x)$

(iv) $E\Delta \equiv \Delta E$

We have,
$$\Delta E f(x) = \Delta[E f(x)]$$
$$= \Delta[f(x+h)]$$
$$= f(x+2h) - f(x+h)$$

and
$$E\Delta f(x) = E[\Delta f(x)]$$
$$= E[f(x+h) - f(x)]$$
$$= E f(x+h) - E f(x)$$
$$= f(x+2h) - f(x+h)$$

$\therefore$ $\Delta E \equiv E\Delta$

(v) $\Delta[f(x)\,g(x)] = f(x)\,\Delta g(x) + g(x+h)\,\Delta f(x)$

L.H.S. $= \Delta[f(x)\,g(x)]$
$$= f(x+h)\,g(x+h) - f(x)\,g(x)$$
$$= g(x+h)f(x) - f(x)\,g(x) + f(x+h)\,g(x+h) - g(x+h)\,f(x)$$
$$= f(x)[g(x+h) - g(x)] + g(x+h)[f(x+h) - f(x)]$$
$$= f(x)\,\Delta g(x) + g(x+h)\,\Delta f(x)$$
$= R.H.S.$

//

(vi)

$$\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,\Delta f(x) - f(x)\,\Delta g(x)}{g(x)\,g(x+h)}$$

L.H.S.

$$\Delta\left[\frac{f(x)}{g(x)}\right]$$

$$= \frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}$$

$$= \frac{f(x+h)\,g(x) - f(x)\,g(x+h)}{g(x)\,g(x+h)}$$

$$= \frac{[g(x)\,f(x+h) - f(x)g(x)] - [f(x)\,g(x+h) - f(x)\,g(x)]}{g(x)\,g(x+h)}$$

$$= \frac{g(x)\,[f(x+h) - f(x)] - f(x)\,[g(x+h) - g(x)]}{g(x)\,g(x+h)}$$

$$= \frac{g(x)\,\Delta f(x) - f(x)\,\Delta g(x)}{g(x)\cdot g(x+h)} = \text{R.H.S.}$$

(vii) $E^0 f(x) = f(x)$

(viii). $E^{-m} f(x) = f(x-mh)$

(xi.) $E^2 f(x) \neq \{E f(x)\}^2$

(ix) $\Delta K = 0$

(x) $EK = K$

when K is a constant

* Relation betn Δ, E, ∇, E^{-1}

(1) $E \equiv 1 + \Delta$

Proof: We have,

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Rightarrow \Delta f(x) = E f(x) - f(x)$$

$$\Rightarrow E f(x) = \Delta f(x) + f(x)$$

$$= (1+\Delta)\, f(x)$$

$$\therefore E \equiv 1 + \Delta$$

(2) $E^2 \equiv (1+\Delta)^2$.

Proof: $E^2 f(x) = f(x+2h)$

$\cancel{= f(x+3h)}$

$= f(x) - 2f(x) + f(x+2h) + 2f(x+h) + f(x) - 2f(x+h)$

$= 2[f(x+h) - f(x)] + [f(x+2h) - 2f(x+h) + f(x)] + f(x)$

$= 2\Delta f(x) + [\{f(x+2h) - f(x+h)\} - \{f(x+h) - f(x)\}] + f(x)$

$= 2\Delta f(x) + [\Delta f(x+h) - \Delta f(x)] + f(x)$

$= 2\Delta f(x) + \Delta[f(x+h) - f(x)] + f(x)$

$= 2\Delta f(x) + \Delta . \Delta f(x) + f(x)$

$= (\Delta^2 + 2\Delta + 1) f(x)$

$= (1+\Delta)^2 f(x)$

$\therefore E^2 \equiv (1+\Delta)^2$

In general, if m is a +ve integer then

$$E^m \equiv (1+\Delta)^m$$

(3) $E^{-1} \equiv 1 - \nabla$.

Proof: $\nabla f(x+h) = f(x+h) - f(x)$

or $\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1} f(x)$

$\Rightarrow E^{-1} f(x) = f(x) - \nabla f(x)$

$\Rightarrow E^{-1} f(x) = (1-\nabla) f(x)$

$\therefore E^{-1} \equiv 1 - \nabla$

(4) $\nabla \equiv E^{-1}\Delta$

Proof: $\nabla f(x) = f(x) - f(x-h)$

$= f(x+h-h) - f(x-h)$

$= E^{-1} f(x+h) - E^{-1} f(x) = E^{-1}\{f(x+h) - f(x)\}$

$$\Rightarrow \nabla f(x) = E^{-1} \Delta f(x)$$

$$\therefore \nabla \equiv E^{-1} \Delta$$