Control limits for $\bar{X}$-charts

Case I: When standards are given, i.e. both μ and σ are known, then the 3- σ control limits for $\bar{X}$ - chart are given by,

$$E(\bar{X}) \pm 3 \text{ S. E. } (\bar{X}) = \mu \pm 3\frac{\sigma}{\sqrt{n}}$$

Case II: When μ and σ are unknown, then the 3- σ control limits for $\bar{X}$- chart are given by,

$$E(\bar{X}) \pm 3 \text{ S. E. } (\bar{X})$$
$$= \bar{X} \pm 3\frac{\bar{R}}{d_2} \cdot \frac{1}{\sqrt{n}}$$
$$= \bar{X} \pm \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R}$$

Since d_2 is a constant depending on n, thus $\frac{3}{\sqrt{n}.d_2}$ also depends only on n and its values have been computed and tabulated for different values of n from 2 to 25.

Thus,

$$UCL = \bar{X} + \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R}$$
$$LCL = \bar{X} - \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R}$$
$$CL = \bar{X}$$

If, on the other hand, the control limits are to be obtained in terms of $\bar{s}$ rather than $\bar{R}$, then an estimate of σ can be obtained from the relation,

$$\bar{s} = C_2 . \sigma$$

or,

$$\sigma = \frac{\bar{s}}{C_2}$$

where,

$$C_2 = \sqrt{2/n} \cdot \frac{\left(\frac{n-2}{2}\right)!}{\left(\frac{n-3}{2}\right)!}$$

Thus,

$$UCL = \bar{X} + \left(\frac{3}{\sqrt{n}.C_2}\right)\bar{S}$$
$$LCL = \bar{X} - \left(\frac{3}{\sqrt{n}.C_2}\right)\bar{S}$$

Control limits for R-chart:

The 3- σ control limits for R-charts are given by

$$E(R) \pm 3\sigma_R$$

E(R) is estimated by $\bar{R}$, and σ_R is estimated from the relation, $\sigma_R = c.E(R) = c.\bar{R}$, where c is a constant depending on n.

Thus,

$$UCL = \bar{R} + 3c.\bar{R} = (1+3c)\bar{R} = D_4\bar{R}$$

$$LCL = \bar{R} - 3c.\bar{R} = (1-3c)\bar{R} = D_3\bar{R}$$

$$CL = \bar{R}$$

The values $D_4 = 1+3c$ and $D_3 = 1-3c$ have been tabulated for different values n from 2 to 25. However, the control limits for R - chart can be obtained directly from the assumed or known value of σ as follows:

$$UCL = D_2.\ \sigma$$

$$LCL = D_1.\ \sigma$$

where D_1, and D_2 have tabulated for values of on from 2 to 25.

Construction of $\bar{X}$ and R control charts:

Control charts are plotted on rectangular coordinate axes, vertical scale representing the statistical measure $\bar{X}$ and R and horizontal scale representing the sample number. Sample points are indicated on the chart by point which may not be joined. If sample points fall within the control limits, LCL and UCL, then it is called that the process is in-control. If the sample points fall outside the control limits, the process is called out of control.

❖ Construct a control chart for mean the range for the following data on the basis of fuses, samples of 5 being taken every hour. Comment on whether the production seems to be under control assuming that these are the first data:

42	42	19	36	42	51	60	18	15	69	64	61
65	45	24	54	51	74	60	20	30	109	90	78
75	68	80	69	57	75	72	27	39	113	93	94
78	72	81	77	59	78	95	42	62	118	109	109
87	90	81	84	78	132	138	60	84	153	112	136

➢ **Control chart for mean ($\bar{X}$)**

$$UCL = \bar{X} + \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R}, \qquad LCL = \bar{X} - \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R}$$

$$CL = \bar{X}$$

➢ **Control chart for Range (R- chart)**

$$\text{UCL}= \bar{R} + 3c.\bar{R} = (1+3c)\bar{R} = D_4\bar{R}, \qquad \text{LCL} = \bar{R} - 3c.\bar{R} = (1-3c)\bar{R} = D_3\bar{R}$$

$$\text{CL}=\bar{R}$$

Calculation:

Sample No. (1)	Sample Observations (2)					Total (3)	Sample mean ($\bar{X}_i$) (4)= (3)÷ 5	Sample Range (R_i) (5)
1	42	65	75	78	87	347	69.4	45
2	42	45	68	72	90	317	63.4	48
3	19	24	80	81	81	285	57.0	62
4	36	54	69	77	84	320	64.0	48
5	42	51	57	59	78	287	57.4	36
6	51	74	75	78	132	410	82.0	81
7	60	60	72	95	138	425	85.0	78
8	18	20	27	42	60	167	33.4	42
9	15	30	39	62	84	230	46.0	69
10	69	109	113	118	153	562	112.4	84
11	64	90	93	109	112	468	93.6	48
12	61	78	94	109	136	478	95.6	75
						Total	859.2	716

From the above data, we get

$$\bar{X} = \frac{1}{k}\sum_{i=1}^{12} \bar{X}_i = \frac{859.2}{12} = 71.6$$

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{12} R_i = \frac{716}{12} = 59.67$$

From the table values for n = 5, $\frac{3}{\sqrt{n}.d_2} = A_2 = 0.577$.

Thus,

$$\text{UCL}= \bar{X} + \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R} = \bar{X} + A_2\bar{R} = 71.6 + 0.577 \times 59.67 = 106.02$$

$$\text{LCL}= \bar{X} - \left(\frac{3}{\sqrt{n}.d_2}\right)\bar{R} = \bar{X} - A_2\bar{R} = 71.6 - 0.577 \times 59.67 = 37.18$$

$$\text{CL}=\bar{X} = 71.6$$

Again, from the table values for n = 5, D_3=0, D_4=2.115 and thus the control limits for R- chart arte given by,

$$\text{UCL}= \bar{R} + 3c.\bar{R} = (1+3c)\bar{R} = D_4\bar{R} = 2.115 \times 59.67 = 126.2$$

$$\text{LCL}= \bar{R} - 3c.\bar{R} = (1-3c)\bar{R} = D_3\bar{R} = 0 \times 59.67 = 0$$

$$\text{CL}=\bar{R} = 59.67$$

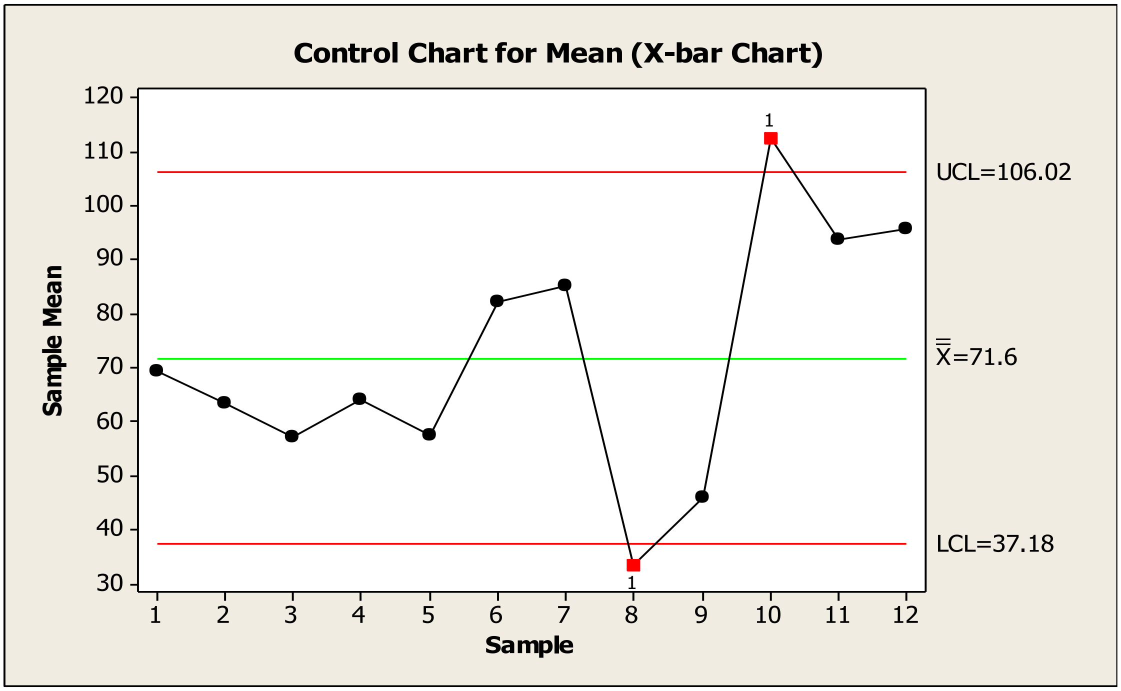

Figure 1: X-bar Chart

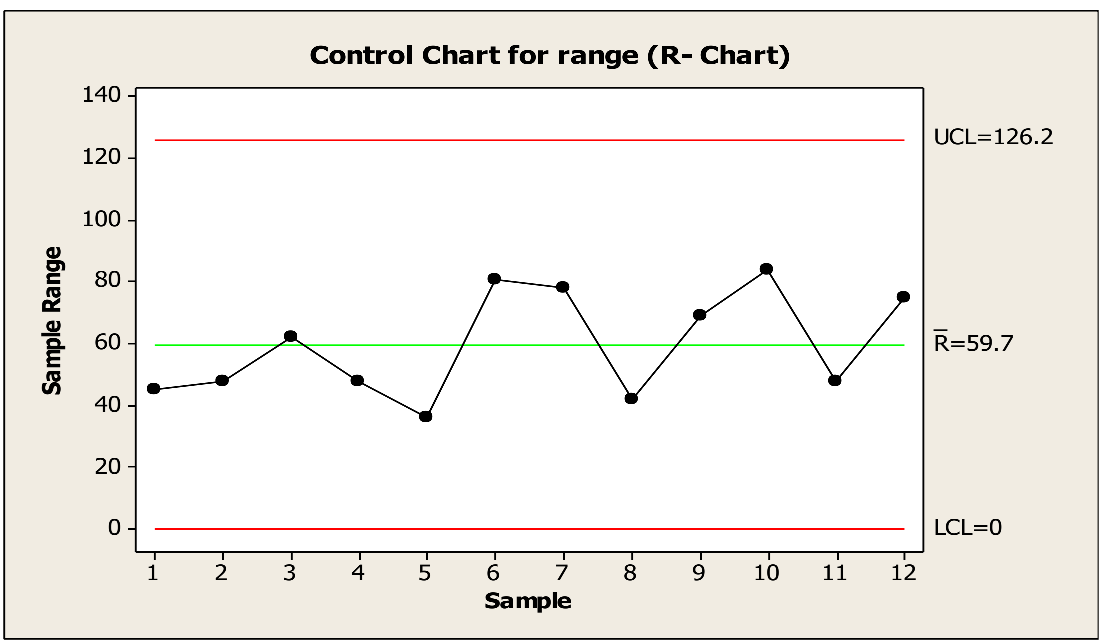

Figure 2: R Chart

Conclusion: From $\bar{X}$ -chart, we see that the sample points 8th and 10th lie outside the upper and lower control limit. So, we conclude that the process is out of control.

Again, From R-chart, we see that all the sample points lie between the upper and lower control limits. Therefore we conclude that the process is under control.

TABLE : FACTORS USEFUL IN THE CONSTRUCTION OF CONTROL CHARTS

	Mean chart			Standard deviation chart					Range chart				
Sample size	Factors for control limits			Factors for central line	Standard deviation chart				Factors for central line	Range chart			
n	A	A_1	A_2	c_2	B_1	B_2	B_3	B_4	d_2	D_1	D_2	D_3	D_4
2	2.121	3.760	1.886	0.5642	0	1.843	0	3.297	1.128	0	3.686	0	3.267
3	1.232	2.394	1.023	0.7236	0	1.858	0	2.568	1.693	0	4.358	0	2.575
4	1.500	1.880	0.729	0.7979	0	1.8080	0	2.266	2.059	0	4.698	0	2.282
5	1.342	1.596	0.577	0.8407	0	1.756	0	2.089	2.326	0	4.918	0	2.115
6	1.225	1.410	0.483	0.8686	0.026	1.711	0.030	1.970	2.534	0	5.078	0	2.004
7	1.134	1.277	0.419	0.8882	0.105	1.672	0.118	1.882	2.704	0.205	5.203	0.076	1.924
8	1.061	1.175	0.373	0.9027	0.167	1.638	0.185	1.815	2.847	0.387	5.307	0.136	1.864
9	1.000	1.094	0.337	0.9139	0.219	1.609	0.239	1.761	2.970	0.546	5.394	0.184	1.816
10	0.949	1.028	0.308	0.9227	0.262	1.584	0.284	1.716	3.078	0.687	5.469	0.223	1.777
11	0.905	0.973	0.285	0.9300	0.299	1.561	0.321	1.679	3.173	0.812	5.534	0.256	1.744
12	0.866	0.925	0.266	0.9359	0.331	1.541	0.354	1.646	3.258	0.924	5.592	0.284	1.716
13	0.832	0.884	0.249	0.9410	0.359	1.523	0.382	1.618	3.336	1.026	5.646	0.308	1.692
14	0.802	0.548	0.235	0.9453	0.384	1.507	0.406	1.594	3.407	1.121	5.693	0.329	1.671
15	0.775	0.816	0.223	0.9499	0.406	1.492	0.428	1.572	3.472	1.207	5.737	0348	1.652
16	0.759	0.788	0.212	0.9523	0.427	1.478	0.448	1.552	3.532	1.285	5.779	0.364	1.636
17	0.0728	0.762	0.203	0.9951	0.445	1.465	0.466	1.534	3.588	1.359	5.817	0.379	1.621
18	0.707	0.738	0.194	0.9576	0.461	1.454	0.482	1.518	3.640	1.426	5.854	0.392	1.668
19	0.688	0.717	0.187	0.9599	0.477	1.443	0.497	1.503	3.689	1.490	5.888	0.404	1.596
20	0.671	0.697	0.180	9.9619	0.491	1.433	0.510	1.499	3.735	1.548	5.922	0.414	1.586
21	0.655	0.679	6.173	0.9638	0.504	1.424	0.523	1.477	3.778	1.606	5.950	0.425	1.575
22	0.640	0.662	0.167	0.9655	0.516	1.415	0.534	1.466	3.819	1.659	5.979	0.434	1.566
23	0.626	0.647	0.162	0.9670	0.527	1.407	0.545	1.455	3.858	1.710	6.006	0.443	1.557
24	0.612	0.632	0.157	0.9684	0.538	1.399	0.555	1.445	3.895	1.759	6.031	0.452	1.548
25	0.600	0.610	0.153	0.9696	0.548	1.392	0.565	1.435	3.931	1.804	6.058	0.459	1.541