
UNIT: 6 TRUTH FUNCTIONS

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6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- I explain the basic symbols used in propositional logic,
- I define the concepts of variable and logical constant,
- I discuss the features of logical constants,
- I describe the scope of logical constants and the use of brackets,
- I analyse the concept of truth-function,
- I describe the truth functions from the concept of function in mathematics,
- I explain the five basic truth-functions with the help of truth-table.

6.2 INTRODUCTION

This unit introduces you to the concepts of variables, logical constants, truth-function and inter-definition of truth-function. All these are very important concepts of propositional logic. With this unit, we shall begin our formal


approach to symbolic logic. As we go through this unit, we shall find that symbolic logic makes use of 'p' and 'q' and other symbols to express its well-formed concepts and formulas.


6.3 PROPOSITIONS: SIMPLE AND COMPOUND

An understanding of the concepts of variables and logical constants necessitates a preliminary understanding of the two kinds of propositions frequently referred to in propositional logic namely, simple and compound proposition. A simple proposition consists of a single statement. It has only one statement as its component part. A compound proposition on the other hand consists of two or more statements as component parts. For example, "you have studied hard" is a simple proposition and "you have studied hard and got 80% marks in Logic" is a compound proposition. In the first example, there is only one statement whereas in the second example, there are two statements.

6.4 Variables

Variables are a type of symbol used in symbolic logic. The lower case letters of English alphabet e.g., p, q, r, s, t, etc., are used as propositional variables to symbolize simple statements. They are thus place-holder for which a statement may be substituted and whose associated value may be changed. For example, in the proposition, "if p then q"; Here p, q are variables. They are symbols which can be replaced by any simple proposition within a definite range. Thus in the above example, the variable 'p' can be replaced by the proposition, "it rains" and 'q' can be replaced by the proposition "crops will grow". Any other propositions can replace the variables.

	ACTIVITY: 6.1
	Is variable a symbol? If so, what does it symbolize? Give some examples.
	Ans:



CHECK YOUR PROGRESS

Q 1: What is propositional logic?
.....

Q 2: What is the basic unit of propositional logic?
.....

Q 3: Fill in the blanks

a) A simple statement consists of a ----- statement.

b) Variables refer to -----individual or object within a specified domain.

c) Compound propositions consist of more than ----- statements as its component parts.

6.5 LOGICAL CONSTANTS

In logic, a logical constant is a symbol that has the same value under every interpretation. Two important types of logical constants are-logical connectives and quantifiers. In propositional logic we are concerned only with the logical connectives which are as follows-

1. Negation sign ' \sim ' (interpreted as 'not' or 'it is false that'...)
2. Conjunction sign ' \cdot ' (interpreted as 'and', or 'both as')
3. Disjunction sign ' \vee ' (interpreted as 'or', 'either...or'...)
4. Conditional sign or the sign of implication ' \supset ' (interpreted as 'if ...then...')
5. Biconditional sign or the sign of equivalence ' \equiv ' (interpreted as 'if and only if...')

Thus using the variables and constants, we can form following compound expressions or logical formulae-

$\sim p$ (read as 'not p' or 'it is false that p')

$p \vee q$ (read as 'p or q')

$p \supset q$ (read as 'if p and q, or 'p implies q')

$p \equiv q$ (read as 'p if and only if q' or p is equivalent to q)

Not (read as 'it is not the case') or the negation is a monadic or unary constant or connective. For, at a time, it can connect only one statement. Some logicians do not accept it as a connective because that it does not serve to

connect two statements. However, it serves to form compound propositions from simple statements by negating the simple statements. In this sense, some other logicians accept it as a connective. It is to be noted that the result of negating any simple proposition is a compound proposition. The other four logical constants or connectives are called dyadic connectives. They are called so because at a time they can connect two statements. The three connectives or constants viz., 'not', 'and' 'or' are also known as the Boolean operators or Boolean connectives. For, these were first used by George Boole, the English logician, as part of his system of logic.

The following table shows the logical constants along with the compound proposition which is formed, the English name of the symbol and the symbol itself:-

Logical Constants	The compound proposition	The English name	Symbol
'not', it is false that	Negative proposition	Tilde, curl	\sim
If-then	Conditional or implicative proposition	Hook, horse-shoe	\supset
and	Conjunctive	Dot	\cdot
If and only if	Equivalent proposition	Equivalence (implies both ways)	\leftrightarrow, \equiv
or	Disjunctive proposition	Vel, wedge	\vee



LET US KNOW

I Propositional logic is an important part of elementary symbolic logic. It is considered to be the starting point of symbolic logic. Propositional logic deals with the logical relationships between propositions without considering the interior structure of the propositions. It analyses the terms, properties, relations and quantifications involved in the proposition. It is also known as the Sentential Logic or the Logic of Truth-functions.

I Propositional formula means an expression or a propositional form which is a combination of symbols according to logical rules.

6.6 THE SCOPE OF LOGICAL CONSTANTS AND THE USE OF BRACKETS

It has been observed that each of the connectives except the 'not' (\sim), connects two and only two propositions, whether these be simple or compound. So if we deal with a proposition in which two compound

propositions are connected, or in which one simple proposition is connected to a simple one, it is essential to make clear which two propositions are connected. This can be known by using brackets.

It has already been stated that every constant except the negation connects two and only two propositions, i.e., the one which immediately precedes the constant and that which follows it, whether these be simple and compound. That is to say, the scope of the constant covers the expression which immediately precedes it, and that which immediately follows it. Thus in the following expression, ---

$$1. \quad p \cdot (q \supset r)$$

The scope of the conjunctive constant i.e., the '·' sign is p (which immediately precedes it) and the whole form enclosed in the round brackets. The scope of the implicative constant i.e., the hook sign is q and r.

Let us consider the following form-

$$2. \quad (p \supset q) \cdot (r \supset s)$$

Here the scope of the first occurrence of the implicative constant is p and q, and the scope of the second hook sign or the implicative constant is r and s. The scope of the conjunctive constant or the dot (·) sign is the compound expression enclosed in the round brackets on either side of it.

In compound propositions, which have other compound propositions as their components, one of the connectives is the main connective. The main connective is that which has the widest scope. In the example no.1, the dot sign is the main connective, as it is also the case in the example no.2. In the following example:-

$$[(p \cdot q) \supset r] \cdot \sim r$$

The second occurrence of the dot sign is the main connective.

q The scope of the negation sign (\sim) :-

The scope of the negation sign is that expression which immediately follows it, whether this is simple or compound. Here are some examples:-

1. $\sim p$, here the scope of the negation sign is 'p'. There is no need for brackets here, because the scope of the negation sign covers a simple proposition only.

$$2. \quad \sim p \supset q$$

Here again, the scope of the negation sign is p. The scope of the hook

sign is $\sim p$ (on the left hand side) and q on the right hand side.

3. $\sim(p \supset q)$

Here the scope of the negation sign is the whole of the expression within the round brackets

4. $\sim[p \supset (q \cdot r)]$

In this example, the scope of the negation sign is $(q \cdot r)$. In example nos. 3 and 4, it is the main constant. In example no.1, it is the main constant. In example no. 2, it is not the main constant. Here the hook sign is the main constant.



CHECK YOUR PROGRESS

Q 4: What do you mean by logical constants? How many logical constants are there?

.....

Q 5: What is the difference between a monadic and a dyadic connective?

.....

Q 6: How many monadic constants or connectives are there in propositional logic?

.....

Q 7: Fill in the blanks-

- a) Every connective connects only ----- propositions
- b) The main connective is that which has the -----scope.
- c) The scope of the negation sign is that expression which immediatelyit.

6.7 TRUTH-FUNCTION

A truth function is a combination of propositions or sentences which has a definite truth-value. As for instance, a conjunction or negation is a truth function whose truth value is determined by the truth values of the components. 'p.q' is a truth function of p and q whose truth value is true when p is true and q is true, and false otherwise. ' $\sim p$ ' is a truth function of p

whose truth value is false when p is true and true when p is false.


Thus there is a correspondence between the truth-values of the truth-functional expression and its component variables. In other words, a compound expression is a function of the truth-values of its simple and atomic components. This is called the truth-functionality of the compound expression. In the case of the truth-functional compound, the value of the whole expression is called output value and values of the individual variables or the simple components are called the input values.

The concept of 'truth-function' is much close to the concept of 'function' in mathematics. To give an example from mathematics, the compound expression, '5+7' is a function because its value is determined by the values of the numbers signified by 5 and 7. Again, let us consider the expression -

$X = 5y + 3$. Here x is a function of y because its value is determined as soon as a value is assigned to the variable y. Hence if the value of y is 5, then the value of x is 28(that is, $x = 5 \times 5 + 3 = 28$). Again, if the value of y is 3, then the value of $x = 5 \times 3 + 3 = 18$.

It may be observed that as in the case of logic where there is a functional correlation between the truth-value of a compound expression and the truth-values of its component variables, so in mathematics also there is a functional correlation between the number signified by a compound arithmetic expression and the numbers signified by its components. However, there are important differences between the concept of function in arithmetic and the concept of truth-function in logic. First, mathematics deals with numerical values such as 1,2,3 etc., whereas symbolic logic deals with truth-values. It is based upon only two truth-values, namely, truth (T) and falsity (F). Second, the range of numerical values in mathematics is infinitely large as there are infinitely many numbers; but the range of truth-values in logic is limited as there are only two truth-values namely (T) and false (F). In symbolic logic, all its statements will be either true or false and, similarly, its compounds will also be either true or false. Hence it is called a bi-valued system.

ACTIVITY 6.2



What is a bi-valued system? Can any statement of propositional logic be neither true nor false? Discuss.

Ans:.....

.....

.....

**CHECK YOUR PROGRESS**

Q 8: What is truth-function? Give examples.

.....

Q 9: Is there any difference between the concepts of logical truth-function and mathematical function?

.....

Q 10: Fill up the blanks-

a) The properties of "true" and "false" are known as ----- which characterize a proposition.

b) Each proposition has -----and only -----truth-value.

6.8 BASIC TRUTH-FUNCTIONS AND THEIR TABULAR REPRESENTATIONS

There are five basic truth-functions in propositional logic.

1. Negative (or contradictory) function- $\sim p$
2. Conjunctive function-- $p \cdot q$
3. Disjunctive function- $p \vee q$
4. Conditional (or implicative) function- $p \supset q$
5. Biconditional (or equivalence) function- $p \equiv q$

These are called basic truth-functions because these are the foundation on the basis of which other forms of more complicated truth-functional expressions are formed

6.9 TRUTH TABLES FOR BASIC TRUTH-FUNCTIONS-

The correspondence between the truth-values of the truth-functional expression and its components can be presented conveniently in the form of tables, which are known as truth- tables. The capital letter 'T' or the numerical '1' is used for 'true' and the capital letter 'f' or the numerical '0' is used for 'false'. Thus, T, F, 1, 0 are the value signs and are written below the component variables of the function. The function is written above the horizontal line in the table. If there is only 1 variable there are just 2 cases of

possible truth-values. With 2 variables there are 4 i.e., $2 \times 2 = 4$ cases. With 3 variables there are 8 i.e., $2 \times 2 \times 2 = 8$ cases. With 4 variables, there are 16 i.e., $2 \times 2 \times 2 \times 2 = 16$ cases. In general, if there are n variables, there must be 2^n cases. We shall now explain the basic truth-functions one by one with the help of truth-tables according to this rule.

I Negative function: ($\sim p$):

The compound statement which is obtained by prefixing a statement say, p with 'not' or 'it is false that' is called a negative function. The negative expression of p is written as $\sim p$.

Let ' p ' be a true statement, then its negation (or denial) ' $\sim p$ ' will be false. Again, if p is false, then ' $\sim p$ ' will be true. Again, if p is true, then ' $\sim p$ ' is false. This can be shown in the truth-table below-:

P	$\sim p$
T	F
F	T

' p ' and ' $\sim p$ ' are contradictions. So they have opposite truth-values. A proposition signified by ' p ' is true if its contradiction is false, and false if its contradiction is true. We may take the truth-table of this function as a definition of the negative function which may be used as a rule for the logical constant occurring in the function.

I Conjunctive function ($p \cdot q$):

The compound proposition obtained by connecting two simple propositions by the logical constant 'and' is called a conjunctive function. If p and q are two simple propositions then the conjunctive function obtained by connecting them with 'and' is written as $p \cdot q$. Both p and q are conjuncts of the conjunctive function $p \cdot q$.

A conjunctive function contains two variables ' p ' and ' q '

A conjunctive function is true only when both the conjuncts are true. In other cases, it is always false. Here we have four possible combinations of truth-values - TT, TF, FT, and FF. The truth-table is as follows-:

p	Q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

We may take the truth table of this function as a definition of the conjunctive function which may be used as a rule for the logical constant occurring in the function.

I Disjunctive Function ($p \vee q$):

The compound proposition obtained by connecting two simple propositions by the constant ' \vee ' (either/or) is called a disjunctive function and is written as $p \vee q$. In logic, the disjunctive $p \vee q$ is interpreted as "either p or q or both". This is inclusive sense of 'either/or', which is accepted, in propositional logic. Both p and q are called 'disjuncts'. The truth-table for the disjunctive function is as follows:-

p	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(There is another sense of 'either/or' in which 'or' means 'either this, or that, but not both'. This is the 'exclusive' sense of 'either/or'. For example, the proposition 'Sankaradeva was born in 1449 or 1514' expresses the exclusive sense of 'either/or' where both the disjuncts cannot be true. The corresponding function is known as 'alternative function'. In propositional logic, the exclusive sense of the 'either/or' is however not accepted).

A disjunctive function is true when at least one of the disjuncts is true and false when both the disjuncts are false. We may take the truth-table of this function as a definition of the disjunctive function which may be used as a rule for the logical constant occurring in the function.

I Implicative or conditional function ($p \supset q$):

If p and q are two statements then the compound expression obtained by connecting them by the constant 'if-then' is called a conditional or implicative function. The conditional or implicative ' $p \supset q$ ' is also known as Material Implication where 'p' is the implicant (antecedent) and 'q' is the implicate (or consequent'). The function $p \supset q$ is false if and only if 'p' is true and 'q' is false. In other cases of truth-value combinations it is true. The truth-table for the implicative function is as follows-

p	Q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

The truth-table of this function may be taken as a definition of the equivalent function which may be used as a rule for the logical constant occurring in the function.


I The Biconditional (or equivalence) function ($p \equiv q$), ($p \equiv q$):

The functions of compound statements of the form "if p then q, and if q then p" are called biconditional (equivalence or equivalent) functions. It is expressed variously in the following ways: 'p if and only if q', 'p is equivalent to q', ' $p \equiv q$ ' and ' $q \equiv p$ ' etc.

A biconditional function $p \equiv q$ is true when its components have the same truth-value, and false when its components differ in truth-value. The following is its truth table:

p	Q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

The truth-table of this function may be taken as a definition of the equivalent function which may be used as a rule for the logical constant occurring in the function.



CHECK YOUR PROGRESS

Q 11: How many basic truth-functions are there in propositional logic? Why are they called basic truth-functions?

.....

Q 12: Write the name of five basic forms of truth-functions.

.....

Q 13: Give example:-

- a) Negative function
- b) Conjunctive function
- c) Implicative function
- d) Disjunctive function
- e) Equivalent function

6.10 INTERDEFINITION OF BASIC TRUTH-FUNCTIONS

It is to be noted that for the purposes of the propositional calculus, we may take the truth-table of each of the truth-functions set out above as a definition of that function. Since the truth-table of a given function is a definition of that function, any other function having the same truth-table will be equivalent to it and may be inter-changed for all logical purposes. For example, the truth-table for ' $\sim p \vee q$ ' is equivalent to the truth-table of ' $p \supset q$ '. It can therefore be regarded as another way of defining the implicative function. Similarly, ' $p \supset q$ ' can be expressed in terms of ' \sim ' and ' \vee ' as ' $\sim(p \cdot \sim q)$ '.

The truth-table is as follows: (table-1)

p	Q	$\sim q$	$\sim(p \cdot \sim q)$
T	T	F	T
T	F	T	F
F	T	F	T
F	F	T	T

Here the final column of the table is identical with the final column of the table for ' $p \supset q$ '. Thus the expression ' $p \supset q$ ', ' $\sim p \vee q$ ' and ' $\sim(p \cdot \sim q)$ ' are logically equivalent and may be for all logical purposes, be substituted one for another. The following tables show the inter definitions of logical functions:-

1. The truth-function ' $p \cdot q$ ' may be defined in terms of ' \sim ' and ' \vee '. This means that they are equivalent. (Table-2)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \cdot q$	$\sim(\sim p \vee \sim q)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

It is seen that the truth-values of the truth-tables ' $p \cdot q$ ' and ' $\sim(\sim p \vee \sim q)$ ' are the same. This means that two functions are equivalent.

2. The truth-function ' $p \supset q$ ' may be defined in terms of ' \sim ' and ' \cdot '. The following table shows that ' $p \supset q$ ' is equivalent to ' $\sim(p \cdot \sim q)$ ' by definition since they have same truth-value. (Table 3)

p	q	$\sim q$	$p \cdot \sim q$	$\sim(p \cdot \sim q)$	$p \supset q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

3. The truth-function $p \cdot q$ is equivalent to $(p \cdot q) \vee (\sim p \cdot \sim q)$; because, as the following table shows, they have the same truth-value. (Table-4)

p	q	$\sim p$	$\sim q$	$p \cdot q$	$\sim p \cdot \sim q$	$p \supset q$	$(p \cdot q) \vee (\sim p \cdot \sim q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

Thus, it can be seen that basic logical constants are interrelated. But ' \sim ' has to be taken as a primitive idea in the real sense of the term; because it is indefinable in terms of any other constants so far introduced.

The following equations sum up the inter definitions among the constants:-

1. $p \cdot q = \text{Df. } \sim(\sim p \vee \sim q)$
 $= \text{Df. } \sim(p \supset \sim q)$
2. $p \vee q = \text{Df. } \sim(\sim p \cdot \sim q)$
 $= \text{Df. } \sim p \supset q$
3. $p \supset q = \text{Df. } \sim p \vee q$
 $= \text{Df. } \sim(p \cdot \sim q)$
4. $p \equiv q = \text{Df. } (p \supset q) \cdot (q \supset p)$
 $= \text{Df. } (p \cdot q) \vee (\sim p \cdot \sim q)$

In the above equations, 'Df' is the abbreviation for 'Definition'. Equation no. 1 shows that the conjunction ' $p \cdot q$ ' can be defined first in terms of negation and disjunction, then secondly in terms of negation and implication.

Equation no. 2 shows that the disjunction ' $p \vee q$ ' can be defined first in terms of negation and conjunction, then in terms of negation and implication. Equation no. 3 shows that ' $p \supset q$ ' can be defined first in terms of negation and disjunction, then in terms of negation and conjunction. Finally, equation no. 4 shows that $p \equiv q$ is equivalent of the conjunction of two conditionals; then, it is expressed in terms of negation, conjunction and disjunction.



CHECK YOUR PROGRESS

Q 14: Answer the following questions-

a) What will be the truth-value of an implicative function when the antecedent and the consequent are both false?

.....

b) What will be the truth-value of a conjunctive function when both the conjuncts are false.

.....

c) Under what condition an implicative function becomes false?

.....

d) Differentiate the inclusive sense and the exclusive sense of disjunction. Which of the two senses is accepted in logic?

.....

e) Under what condition an equivalent function becomes true?

.....



6.11 LET US SUM UP

- I Logical constants and variables are the basic symbols used in propositional logic. The constants are those symbols, which express the form of the compound propositions and maintain the same meaning throughout every occurrence in the propositional formulae. They have fixed meaning. Every logical constant is also known as a connective because they form compound propositions from simple or other compound propositions by connecting them. Each connective connects two and only two propositions. The constant '~' (negation) is not a connective in this sense according to some logicians. At any rate, it forms a compound proposition simply by denying or negating a simple proposition. In compound propositions which have other compound propositions as their components, one of the connectives is the main connective. The main connective is that which has the widest scope.
- I A variable on the other hand, is a symbol which can stand for any one

of a given range of values within a domain. Lower case letters of English alphabet p, q, r, s, t are used as propositional variables which represent simple statements in expressions which contain several statements. They are called variables because like the symbols x and y in algebra, they appear in propositional forms of formulae and they could be replaced by any member of a certain range. Thus while variables may assume any meaning of a definite range, the constant maintain the same meaning throughout every occurrence in propositional formulae.

- I Another very important notion of symbolic logic is the notion of truth-function. A truth-function is a compound expression where the value of the expression is determined by the truth-value of the constituent variables. In other words, the value of the compound expression is determined as soon as the truth-values of the constituent variables are known. For example, the statement 'pvq' is a truth-function, because the truth-value of the expression is determined by the truth-values of the constituent variables 'p' and 'q'.
- I There are five basic truth-functions namely-(a) Negative function, (b) Conjunctive function, (c) Disjunctive function, (d) Implicative function, and (e) Biconditional function. These five basic truth-functional expressions may be defined with the help of truth-table which is a tabular order on which all possible truth-values of the compound statements are displayed, through the display of the truth-values of their simple components. The truth-table of each of the basic function is taken as a definition.
- I It has also been observed that basic logical constants are inter-related. Since the truth-table of a given function is a definition of that function, any other function having the same truth-table will be equivalent to it and may be inter-changed for all logical purposes.



6.12 FURTHER READINGS

- 1) Irving M. Copi and Carl, Cohen : Introduction to Logic, Prentice-Hall India, Eleventh Edition
- 2) A.H. Basson and D.J.O'Connor : Introduction to Symbolic Logic. Oxford University Press
- 3) Dr. Shyam kishor Sing : Modern Logic (vol. 1.) Jalukbari, Guwahati-14.



6.13 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Propositional logic is an important part of elementary symbolic logic, which deals with the logical relationships holding between the propositions without considering the interior structure of propositions. It is also known as the Sentential Logic or the logic of Truth-functions.

Ans to Q No 2: A simple statement is the basic unit of propositional logic.

Ans to Q No 3: a) Single b) unspecified c) one

Ans to Q No 4: The logical constants are those symbols, which express the form of the compound propositions and maintain the same meaning throughout every occurrence in the propositional formulae. There are five logical constants: ' \supset ' (implication), ' \vee ' (disjunction), ' \cdot ' (conjunction), ' \equiv ' (equivalence), ' \sim ' (negation).

Ans to Q No 5: A monadic connective connects only one statement at a time whereas a dyadic connective connects two statements at a time. ' \sim ' (negation) is an example of monadic constant. On the other hand, ' \supset ' (implication), ' \vee ' (disjunction), ' \cdot ' (conjunction), ' \equiv ' (equivalence) are called the dyadic constants.

Ans to Q No 6: There is one monadic constant or connective in propositional logic.

Ans to Q No 7: a) two, b) widest c) follows.

Ans to Q No 8: A truth-function is a compound expression where the value of the expression is determined by the truth-value of the constituent variables; the statement or the compound expression is determined as soon as the truth-values of the constituent variables are known. For example, the statement, ' $p \vee q$ ' is a truth-function, because the truth-value of the expression is determined by the truth-values of the two constituent variables ' p ' and ' q '. Similarly, ' $p \cdot q$ ', ' $[(p \cdot q) \cdot (p \vee \sim q)]$ ', ' $(p \vee q) \cdot (q \vee p)$ ' etc. are some examples of truth-functions.

Ans to Q No 9: Though the concept of truth-function in logic is derived from the notion of function in mathematics, yet there are differences between the two. First, mathematics deals with numerical values such as 1,2,3 etc., whereas symbolic logic deals with truth-values. It is based upon only two truth-values, namely, truth (T) and falsity (F). Second, the range of numerical values in mathematics is infinitely large as there are infinitely many numbers;

but the range of truth-values in logic is limited as there are only two truth-values namely (T) and false (F). In symbolic logic, all its statements will be either true or false and, similarly, its compounds will also be either true or false. Hence it is called a bi-valued system.

Ans to Q No 10: a) truth-value b) one and only one.

Ans to Q No 11: There are five basic truth-functions in propositional logic. They are called basic truth-functions because on the basis of these truth-functional expressions, other more compound forms of truth-functional expressions are formed.

Ans to Q No 12: The name of five basic truth-functional expressions are - (1) Negative function, (2) Conjunctive function, (3) Disjunctive function, (4) Implicative function, (5) Equivalent function.

Ans to Q No 13 : \sim , \cdot , \vee , \supset .

Ans to Q No 14: a) Ans: true (b) Ans: false

c) Ans: When the antecedent is true and the consequent is false.

d) Ans: In the inclusive sense of 'either \ or' 'or' means "either this or that or both" On the other hand, in the exclusive sense, 'or' means 'either this or that but not both'. In propositional logic, the inclusive sense is accepted.

(e) Ans: An equivalent function becomes true only when both the components have the Same truth-value.



6.14 MODEL QUESTIONS

A Very short questions

Q 1: Define truth function.

Q 2: What is variable?

Q 3: What is implicative function?

Q 4: What do you mean by logical constant?

Q 5: Give an example of simple proposition.

Q 6: What is compound proposition?

Q 7: What is a bi-valued system?

Q 8: What is truth-table?

Q 9: How many kinds of truth-values are there in symbolic logic?

Q 10: What are the five basic truth functions?

B. Short questions (Answer in about 150 words)

Q 1: Write a short note on truth function.

Q 2: Write is truth table? Explain briefly

Q 3: What is truth function? Explain in brief.

C. Long questions (Answer in about 300-500 words)

Q 1: What is truth-function? Differentiate the concept of truth-function in logic from the concept of function in mathematics.

Q 2: What are the basic truth-functions? Explain each of them with the help of truth-tables.

Q 3: Define truth-function. Explain implicative and the equivalent truth-function by constructing truth-tables for them.

Q 4: Explain the interdefinition of basic truth functions with examples.
