※ Exact differential eqⁿs and integrating factors:

The first-order differential eqⁿs may be expressed in either the derivative form

$$\frac{dy}{dx} = f(x,y) \quad \text{-----} (1)$$

or the differential form

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \text{---} (2)$$

An eqⁿ in one of these forms may readily be written in the other form. For example, the eqⁿ

$$\frac{dy}{dx} = \frac{x^2+y^2}{x-y}$$ is of the form (1).

It may be written

$$(x^2+y^2)\,dx + (y-x)\,dy = 0$$, which is of the form (2). The eqⁿ $(\sin x + y)\,dx + (x+3y)\,dy = 0$, which is of the form (2), may be written in the form (1) as $\frac{dy}{dx} = -\frac{(\sin x + y)}{(x+3y)}$

In the form (1), it is clear ~~that~~ from the notation itself that y is regarded as the dependent variable and x as the independent one; but in the form (2) we may actually regard either variable as the dependent one and the other as the independent. However, in this text, in all differential eqⁿs of the form (2) in x and y, we shall regard y as dependent and x as independent, unless the contrary is specifically stated.

Def^n: Let F be a function of two real variables such that F has continuous first partial derivatives in a domain D. The total differential dF of the function F is defined by the formula

$$dF(x,y) = \frac{\partial F(x,y)}{\partial x}\, dx + \frac{\partial F(x,y)}{\partial y}\, dy \quad \text{for all } (x,y) \in D.$$

Eg: Let F be the function of two real variables defined by

$$F(x,y) = xy^2 + 2x^3 y \quad \text{for all real } (x,y). \text{ Then}$$

$$\frac{\partial F(x,y)}{\partial x} = y^2 + 6x^2 y, \quad \frac{\partial F(x,y)}{\partial y} = 2xy + 2x^3$$

and the total differential dF is defined by

$$dF(x,y) = (y^2 + 6x^2 y)\, dx + (2xy + 2x^3)\, dy$$

for all real (x,y)

Def^n: The expression $M(x,y)\,dx + N(x,y)\,dy$ ---- (3)

is called an exact differential in a domain D if there exists a function F of two real variables such that this expression equals the total differential $dF(x,y)$ for all $(x,y) \in D$. That is, expression (3) is an exact differential in D if there exists a function F such that

$$\frac{\partial F(x,y)}{\partial x} = M(x,y) \quad \text{and} \quad \frac{\partial F(x,y)}{\partial y} = N(x,y)$$

for all $(x,y) \in D$.

If $M(x,y)\,dx + N(x,y)\,dy$ is an exact differential, then the differential equation

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

is called an exact differential equation.