Ex If $a_1, a_2, \cdots, a_n$ are any real numbers, then

$$|a_1+a_2+\cdots+a_n| \leq |a_1|+|a_2|+\cdots+|a_n|.$$

Soln: Since $|a_1+a_2| \leq |a_1|+|a_2|$

and $|a_1| \leq |a_1|$

So the result is true for $n = 1, 2$.

Let the result be true for $n = k$

$\therefore |a_1+a_2+\cdots+a_k| \leq |a_1|+|a_2|+\cdots+|a_k|$

Now,

$$|a_1+a_2+\cdots+a_k+a_{k+1}| \leq |a_1+a_2+\cdots+a_k|+|a_{k+1}|$$

$$\leq |a_1|+|a_2|+\cdots+|a_k|+|a_{k+1}|$$

Thus the result is true for $n = k+1$.

So by principle of Mathematical induction the result is true for all n.

$$\therefore |a_1+a_2+\cdots+a_n| \leq |a_1|+|a_2|+\cdots+|a_n|$$

Ex: Determine the set A of $x \in \mathbb{R}$ such that $|2x+3| < 7$.

Soln: We know, if $c \geq 0$ then $|a| \leq c \iff -c \leq a \leq c$

Taking $c = 7$, $a = 2x+3$

$|2x+3| < 7 \Rightarrow -7 < 2x+3 < 7$

$\Rightarrow -7-3 < 2x < 7-3$

$\Rightarrow -10 < 2x < 4 \Rightarrow -5 < x < 2$

$\therefore A = \{x \in \mathbb{R} \mid -5 < x < 2\}$

Ex. Determine the set $B = \{x \in \mathbb{R} : |x-1| < |x|\}$

Soln: Case 1. When $x \geq 1$ then $x - 1 \geq 0$.

$\therefore |x-1| = x-1$ and $|x| = x$

$\therefore |x-1| < |x| \Rightarrow x - 1 < x$, which is always true.

$\therefore$ all x such that $x \geq 1$ belongs to the set B.

Case 2. When $0 \leq x < 1$. then $x - 1 < 0$

$\therefore |x-1| = -(x-1)$ and $|x| = x$

$\therefore |x-1| < |x| \Rightarrow -(x-1) < x \Rightarrow 1 < 2x \Rightarrow x > \frac{1}{2}$

Thus this Case contributes all x such that $\frac{1}{2} < x < 1$.

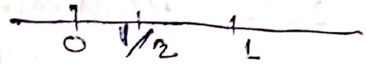
Case 3. When $x < 0$. Then $x - 1 < 0$

$\therefore |x-1| = -(x-1)$ and $|x| = -x$

$\therefore |x-1| < |x| \Rightarrow -(x-1) < -x \Rightarrow -x+1 < -x$

$\Rightarrow 1 < 0$, which is impossible

So no values of x satisfying $x < 0$ satisfy the given inequality.



$\therefore B = \{x \in \mathbb{R} : x \geq 1\} \cup \{x \in \mathbb{R} : \frac{1}{2} < x < 1\}$

$= \{x \in \mathbb{R} : x > \frac{1}{2}\}$

#

Ex: Let the function f be defined by $f(x) = (2x^2+3x+1)/(2x-1)$ for $2 \leq x \leq 3$. Find a Constant M such that $|f(x)| \leq M$, for all x satisfying $2 \leq x \leq 3$.

Soln $f(x) = \dfrac{2x^2+3x+1}{2x-1} \Rightarrow |f(x)| = \left|\dfrac{2x^2+3x+1}{2x-1}\right| = \dfrac{|2x^2+3x+1|}{|2x-1|}$

From triangle inequality, $|2x^2+3x+1| \leq 2.|x^2| + 3.|x| + |1|$

$= 2|x|^2 + 3|x| + 1$

$\leq 2.3^2 + 3.3 + 1 = 28$

$[\because x \leq 3]$

Also, $|2x-1| \geq 2|x| - 1 \geq 2.2-1 = 3$ [$\because |x| \geq 2$ as $x \geq 2$

$\therefore$ for $2 \leq x \leq 3$, we have

$$|f(x)| = \frac{|2x^2+3x+1|}{|2x-1|} \leq \frac{28}{3}.$$

Taking $M = \frac{28}{3}$ we get

$|f(x)| \leq M$ for all x, $2 \leq x \leq 3$.

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