* Complete primitive (or general solution), particular solution and singular solution:

Let $F(x, y, y_1, y_2, y_3, \ldots, y_n) = 0$ --- (1)

be an nth order ordinary differential equation.

(i) A solution of (1) containing n independent arbitrary constants is called a general solution.

(ii) A solution of (1) obtained from a general solution of (1) by giving particular values to one or more of the n independent arbitrary constants is called a particular solution of (1).

(iii) A solution of (1) which cannot be obtained from any general solution of (1) by any choice of the n independent arbitrary constants is called a singular solution of (1).

Example: $y = c_1 e^{x} + c_2 e^{2x}$ ------ (2)

is the general solution of $y'' - 3y' + 2y = 0$ --- (3)

[$\because$ $y' = c_1 e^{x} + 2c_2 e^{2x}$

$y'' = c_1 e^{x} + 4c_2 e^{2x}$

$= 3c_1 e^{x} + 6c_2 e^{2x} - 2c_1 e^{x} - 2c_2 e^{2x}$

$= 3(c_1 e^{x} + 2c_2 e^{2x}) - 2(c_1 e^{x} + c_2 e^{2x})$

$= 3y' - 2y$

$\Rightarrow y'' - 3y' + 2y = 0$]

Since c_1 and c_2 are independent arbitrary constants and the order of (3) is two, (2) is a general solution of (3). Some particular solutions of (3) are given by $y = e^x + e^{2x}$, $y = e^x - 2e^{2x}$ etc.

The equation of the form $y = px + f(p)$ — (4)

where, $p = \frac{dy}{dx}$ and $f(p)$ is a function of p is called as Clairaut's equation.

Differentiating eq^n (4) w.r.t x both sides, we get

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\Rightarrow \quad p = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\Rightarrow \quad [x + f'(p)]\frac{dp}{dx} = 0$$

$\Rightarrow$ either $\frac{dp}{dx} = 0$ ---- (5)

or $x + f'(p) = 0$ ---- (6)

Eq^n (5) gives $p = c$

Putting the value of p in eq^n (4), we get

$$y = cx + f(c) \quad \text{----} \quad (7)$$

which is called the general solution of the differential eq^n (4).

Eliminating p from eq^{n}s (4) and (6), we get relation between x and y called singular sol^n of the differential equation.