Unit - 2

Finite differences:

Suppose we have a function $y = f(x)$, where x can take the values $a, a+h, a+2h, \dots, a+nh$. Let the corresponding values of y i.e. $f(x)$ be $f(a), f(a+h), f(a+2h), \dots, f(a+nh)$. The expression $f(a+h) - f(a)$ is called the first difference of $f(a)$ and we denote this difference by $\Delta f(a)$.

Thus $\Delta f(a) = f(a+h) - f(a)$

Similarly $f(a+2h) - f(a+h), \dots, f(a+nh) - f\{a+(n-1)h\}$ are all called the first differences and generally denoted as $\Delta f(x) = f(x+h) - f(x)$, $x = a, a+h, \dots, a+(n-1)h$. ---- (1)

Here Δ is an operator and is called a forward difference operator, h is called the interval of differencing.

If we operate Δ on (1), we get second differences of the function values.

$$\Delta\{\Delta f(x)\} = \Delta\{f(x+h) - f(x)\}$$

$$\Rightarrow \Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

$$\Rightarrow \Delta^2 f(x) = \{f(x+2h) - f(x+h)\} - \{f(x+h) - f(x)\}$$

$$= f(x+2h) - 2f(x+h) + f(x)$$

Here Δ^2 represents that the operation of difference has been done twice.

We may continue to define third difference as

$$\Delta^3 f(x) = \Delta\{\Delta^2 f(x)\} = \Delta\{f(x+2h) - 2f(x+h) + f(x)\}$$

$$= \Delta f(x+2h) - 2\Delta f(x+h) + \Delta f(x)$$

$$= \{f(x+3h) - f(x+2h)\} - 2\{f(x+2h) - f(x+h)\} +$$

$\{f(x+h) - f(x)\}$

$$\Delta^3 f(x) = f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)$$

and so on.

In general the mth forward difference is given by

$$\Delta^m f(x) = \Delta^{m-1} f(x+h) - \Delta^{m-1} f(x)$$

※ Difference table:

Argument x	Entry $f(x)$	First differences $\Delta f(x)$	Second differences $\Delta^2 f(x)$
a	$f(a)$		
		$f(a+h) - f(a) = \Delta f(a)$	
$a+h$	$f(a+h)$		$\Delta f(a+h) - \Delta f(a) = \Delta^2 f(a)$
		$f(a+2h) - f(a+h) = \Delta f(a+h)$	
$a+2h$	$f(a+2h)$		$\Delta f(a+2h) - \Delta f(a+h) = \Delta^2 f(a+h)$
		$f(a+3h) - f(a+2h) = \Delta f(a+2h)$	
$a+3h$	$f(a+3h)$		$\Delta f(a+3h) - \Delta f(a+2h) = \Delta^2 f(a+2h)$
		$f(a+4h) - f(a+3h) = \Delta f(a+3h)$	
$a+4h$	$f(a+4h)$		
⋮	⋮	⋮	⋮

Ex) Given $f(0)=3$, $f(1)=12$, $f(2)=81$, $f(3)=200$, $f(4)=100$ and $f(5)=8$. Form a difference table and find $\Delta^5 f(0)$.

Solⁿ:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	3					
		9				
1	12		60			
		69		-10		
2	81		50		-259	
		119		-269		
3	200		-219		496	
		-100		227		
4	100		8			
		-92				
5	8					

755

$\therefore \Delta^5 f(0) = 755$

Ex) Qr

x :	1	2	3	4	5
y :	2	5	10	20	30

find by forward difference table $\Delta^4 y(1)$

Ans:

$\Delta^4 f(1) = -8$