Triangle Inequality: If $a, b \in \mathbb{R}$, then
$$|a+b| \le |a|+|b|$$

Pf:- For all $a \in \mathbb{R}$, we know
$$-|a| \le a \le |a|$$
Ily, for $-|b| \le b \le |b|$
Adding these inequalities, we obtain
$$-|a|-|b| \le a+b \le |a|+|b|$$
$$\Rightarrow -(|a|+|b|) \le a+b \le |a|+|b|$$
$$\Rightarrow |a+b| \le |a|+|b|$$

Corollary. If $a, b \in \mathbb{R}$, then
(a) $\big||a|-|b|\big| \le |a-b|$
(b) $|a-b| \le |a|+|b|$

Pf:-
$$a = a-b+b$$
$$\Rightarrow |a| = |(a-b)+b| \le |a-b|+|b|$$
$$\Rightarrow |a| \le |a-b|+|b|$$
$$\Rightarrow |a|-|b| \le |a-b| \quad \text{— (I)}$$

Similarly, $b = b-a+a$
$$\Rightarrow |b| = |(b-a)+a| \le |b-a|+|a|$$
$$\Rightarrow -|b-a| \le |a|-|b| \quad \text{— (II)}$$
But $|a-b| = |-(a-b)|$ $\quad$ [$\because |a| = |-a| \ \forall a \in \mathbb{R}$]
$$= |b-a|$$
$$\Rightarrow -|a-b| = -|b-a|$$
(II) $\Rightarrow -|a-b| = -|b-a| \le |a|-|b|$
$$\Rightarrow -|a-b| \le |a|-|b| \quad \text{— (III)}$$

Combining (i) and (iii), we get

$$-|a-b| \leq |a| - |b| \leq |a-b|$$

$$\Rightarrow \big| |a| - |b| \big| \leq |a-b|$$

(b) From triangle inequality, we get

$$|a+b| \leq |a| + |b|, \text{ for all } a, b \in \mathbb{R}.$$

Replacing b by $-b$, we get

$$|a+(-b)| \leq |a| + |-b|$$

$$\Rightarrow |a-b| \leq |a| + |b| \quad [\because |b| = |-b|]$$