UNIT 1: FORMAL PROOF OF VALIDITY

UNIT STRUCTURE

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1.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- Define the method of deduction
- Define the formal proof of validity
- Explain the strategy of deduction
- Discuss the rules of inference
- Describe the rules of replacement
- Distinguish between replacement and substitution
- Explain the test of formal proof

1.2 INTRODUCTION

This unit introduces to you the formal proof of validity. Truth tables provide an effective method for demonstrating the validity of arguments. In theory, truth tables are adequate to test the validity of any argument of general type. But, in practice, truth tables become unwieldy and tiresome when the number of component statements increases. Thus, the truth table method is inconvenient for testing the validity of arguments containing a large number of statements. A more convenient method for establishing the validity of arguments is the Method of Deduction or what may be called the method of Derivation by Substitution. The method of deduction is applicable to those arguments which are valid. In this method we simply construct a proof of their validity in order to construct a formal proof of their validity. We have to apply certain rules to the given premises and go on deducing consequences from them unless and until we get the original conclusion. Since in this method we deduce conclusion from the original premises to get the original conclusion through the application of certain rules, thereafter, this method is called the method of deduction.

This deductive procedure is called formal; because, it relies on the valid argument forms to show as to how the conclusion can be deduced from the given set of premises. The valid argument forms are used as the logical rules to determine the consequences which can be validity inferred from the premises.

As we all know that there is a distinction between argument and argument form. If an argument form is valid then any argument of that form will always be valid. An elementary valid argument is an argument which is valid.

1.3 FORMAL PROOF OF VALIDITY

A formal proof of validity for a given argument may be defined “to be a sequence of statements, each of which is either a premiss of that argument or follows from preceding statements by an elementary valid argument and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved.” (I.M. Copi.)

A substitution instance of an elementary argument forms.

$(F \supset \sim G) \supset (\sim H V \sim I)$

F ⊃ ~ G

∴ ~ H V ~ I

Is an elementary valid argument; because, it is a substitution instance of the elementary valid argument form, Modus Ponens (M.P). It results from

P ⊃ Q

P

∴ q

by substituting (F ⊃ ~ G) for P and (~ H V ~ I) for q.

Therefore, it is of that form even though Modus Ponens is not the specific form of the given argument.

1.4 STRATEGY FOR DEDUCTION

The formal proof of validity of an argument can be constructed easily. The strategy for deduction will be as follows:

a) For constructing the validity of an argument formulated in ordinary language, the statements of the argument will be symbolized by using the capital letters of the alphabet, to bring out the logical form of the argument.

b) This proof of validity will be started by stating and listing the given premises of the argument in one column.

c) All the premises and the statements deduced from them must be numbered serially and must be put in on column and the conclusion must be separated from the premises and it must be written to the right of the last. Premise separated by a slanting line with the symbol '∴' which automatically marks of all the statements above it to be premises.

d) The statements that are deduced from the original premises by the application of rules must be put along with the given premises with "justifications" written beside then. The justification species the statement from which and the rule by which the statement in question is deduced.

e) The statements that are deduced from the original premises by the application of rules are to be taken as premises and the deduction must continue until we get the original conclusion.

Now let us take an argument and see how the formal proof of validity of the argument can be constructed.

1. (F V (G V H)

2. (G ⊃I). (H ⊃ J)
3. (I ⊃ J) ⊃ (F V H)
4. ~F / ∴H
5. (G V H) 1, 4 D.S
6. I V J 2, 5 C.D
7. F V H 3, 6 M.P
8. H 7, 4 D.S

CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false.

a) The method of deduction is applicable only to those arguments which are valid. (True/False)

b) The method of deduction is more convenient than the truth table method for establishing the validity of arguments. (Truth/False)

c) The method of deduction is also known as the method of derivation by substitution. (Truth/False)

Q 2: Fill in the blanks

a) Rules of inference and ……………………..are the two sets of rules of derivation.

b) Rules of replacement consists only of……………………………..

c) Derivation rules is also sometimes called………………………

Q 3: What is the Method of Deduction?

Q 4: Why is this derivation procedure called formal?

Q 5: How do you define a formal proof of validity for a given argument?

Q 6: What is an elementary valid argument?

Q 7: What does the justification in terms of the elementary valid argument form or the logical rule for a new premise of the argument specify?

1.5 DERIVATION RULES: RULES OF INFERENCE

Rules of Inference and Rules of Replacement are the two sets of rules for derivation. Rules of inference are nothing but some valid argument forms whose validity is established by truth tables. These rules are used in constructing formal proof of validity. The following rules are the Rules of inference.

Rules of Inference:

1. Modus Ponens (M.P)

   $p \supset q$

   p

   $\therefore q$

2. Modus Tollens (M.T)

   $p \supset q$

   $\sim q$

   $\therefore \sim p$

3. Hypothetical Syllogism (H.S)

   $p \supset q$

   $q \supset r$

   $\therefore p \supset r$

4. Disjunctive Syllogism

   $p \vee q$

   $\sim p$

   $\therefore q$

5. Constructive Dilemma (C.D)

   $(p \supset q)\ (r \supset s)$

   $p \vee r$

   $\therefore q \vee s$

6. Destructive Dilemma

   $(p \supset q)\ (r \supset s)$

   $\sim q \vee \sim s$

   $\therefore \sim p \vee \sim r$

7. Simplification (Simp)

   $p.\ q$

   $\therefore p$

8. Conjunction (Conj)

   p

   q

   $\therefore p.q$

9. Addition (Add)

   p

   $\therefore p \vee q$

10. Absorption (Abs)

   $p \supset q$

   $p \supset (p.q)$

These rules of inference correspond to elementary argument forms, the validity of which is established easily by truth tables. These rules can be used to construct formal proofs of validity for more complicated arguments.

1.6 DERIVATION RULES

Rules of inference are not regarded as sufficient for constructing the validity of many other arguments. So additional rules are required in such cases. As it is known, here only truth functional compound statements concern us. Hence "if any part of a compound statement is replaced by any logically equivalent expression to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This is sometimes called the rule of replacement and sometimes the principle of extensionality. The rule of Replacement is adopted as an additional principle of inference.

The rule of replacement consists only of logical equivalences. For example, one of rules of replacement is De Morgan's theorem that is ~(P.Q) ≡ ~p v ~q. We can infer one from the other since they are logically equivalent. Therefore, ~ (p.q) we can come to ~p v ~ Q

The list of replacement rules with their complete and abbreviated names is presented as follows:

Rules of Replacement

11. De Morgan's Theorems (De. M)
    ~ (p. q) ≡ (~ p v ~ q)
    ~ (p v q) ≡ (~ p. ~ q)

12. Commutation (Com)
    (p v q) ≡ (q v p)
    (p.q) ≡ (q. p)

13. Association (Assoc)
    [p v (q v r)] ≡ [(p v q) vr)]
    [p. (q . r)] ≡ [(p. q) . r]

14. Distribution (Dist)
    [p. (q V r)] ≡ [(p.q) v (p. r)]
    [p v (q. r) ≡ [(p v q) . (p v r)]

15. Double Negation (D.N.)
    p ≡ ~~ p

16. Transposition (Trans)
    (p ⊃ q) ≡ (~q ⊃ ~ p)

17. Material Implication (Impl)
    (p ⊃ q) ≡ (~p v q)

18. Material Equivalence (Equiv.)
    (p ≡ q) ≡ [(p ⊃ q). (q ⊃ p)]
    (p ≡ q) ≡ [(p. q) v (~ q. ~ p)]

19. Exportation (Exp.)
    [(p. q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]

20. Tautology (Taut)
    p ≡ (p v p)
    p ≡ (p. p)

Each of these rules is stated as equivalences with ≡ as the main connective. This indicates that we can infer the left hand side of the ≡from the right hand side and vice versa.

1.7 DIFFERENCES BETWEEN REPLACEMENT AND SUBSTITUTION

The process of replacement is very different from that of substitution. The two important differences between replacement and substitution are the following:

1 In replacement, one statement can be replaced only by a statement logically equivalent to it where the logical equivalence of the two statements is given in rules of replacement.

On the other hand, in substituting statements for statement variables in an argument form, any statement variable can have any statement substituted for it.

2 In substitution, every occurrence of a statement variable in an argument form must have the same statement substituted for it.

However, in replacement, one or several occurrences of a statement may be replaced by a logically equivalent statement, without other occurrences of that statement having to be replaced also.

1.8 RULES OF INFERENCE AND RULES OF REPLACEMENT

There are some major differences between rules of inference and rules of replacement.

Rules of Inference can be applied only to whole lines of proof. For example, A can be inferred from A.B. by simplification only if A.B constitutes a whole line. But, neither A nor (A ⊃ C) follows from (A.B) ⊃ C by simplification or by any other Rules of Inference.

On the other hand, rules of replacement can be applied either to whole lines or to parts of lines. For example, the statement A ⊃ (B ⊃ C) can be inferred from the whole line (A. B) ⊃ C by Exportation. Again, the statement [A ⊃ (B ⊃ C)] V D can be inferred from [(A .B) ⊃ C) VD by Exportation.

Thus, the rules of replacement can be used even where they do not constitute whole lines of a proof. But, the Rules of Inference can be used only with whole lines of a proof serving as premises. Moreover, in contrast to the rules of inference, the rules of replacement authorize replacement of a statement by its specified logically equivalent statement. Again, the Rules of Inference work in one direction while Rules of replacement work in both directions. Replacement in both directions is indicated by the symbol '≡ '

1.9 TEST OF FORMAL PROOF

The test of whether or not a given sequence of statements is a formal proof, is effective. Direct observation shows whether or not, every line beyond the premises actually follows from preceding lines by one of the given rules of inference. Thus, it can be decided mechanically in a finite number of steps whether or not a given sequence of statement constitutes a formal proof with reference to the given list of rules of inference. Thinking is not required in any case neither in thinking about what the statements mean, or in using logical intuition to check the validity of the deduction of any line. There is a finite, mechanical procedure for deciding whether or not the deduction is legitimate even where the justification of a statement is not written beside it. Each line is preceded by only a finite number of other lines, and only a finite number of Rules of Inference are adopted. Though time consuming, it can be verified by inspection whether the line in question follows from any single preceding line or from any pair of preceding lines by any one of the rules of inference listed. Again, the any legitimacy line can be decided by a finite number of observations none of which involves anything more than comparing shapes or patterns. To preserve this effectiveness, the rule is that only one Rule of Inference should be applied at a time. Strictly speaking, the explanatory notation beside each statement is not a part of the proof. However, it is helpful and should always be included. Thus, it is known that the test of whether or not a given sequence of statements is a formal proof is effective. But, constructing such a formal proof is not an effective procedure. The use of truth tables is completely mechanical. But in constructing a formal proof of validity on the basis of the rules of inference, it is necessary to think or 'figure out' where to begin and how to proceed. There is no effective or purely mechanical method of procedure. But, it is usually much easier to construct a formal proof of validity than to write out a truth table.

ACTIVITY 1.1

Do you think that Rules of Inference are sufficient for proving the validity of arguments?

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

CHECK YOUR PROGRESS

Q 8: How does replacement differ from substitution?

Q 9: What are the differences between rules of inference and rules of replacement?

Q 10: How do you know that test of formal proof is effective?

Q 11: How do you determine that constructing a formal proof is not an effective procedure?

Q 12: State whether the following statements are true or false

a) The Rules of Inference work in one direction. (True / False)

b) The Rules of Inference are regarded as sufficient for proving the validity of arguments. (True / False)

C) The rules of replacement work in both directions. (True / False)

d) The use of truth tables is completely mechanical.(True /False)

1.10 GENERAL SUGGESSTIONS FOR FORMAL DEDUCTION

As we observe, no purely mechanical rules are there for constructing formal proofs. But, some general suggestions or hints are found for doing derivation.

The first is to begin the process by deducing conclusions from the given premises with the help of the given rules. When more and more of these sub conclusions become available as the premises for further deduction, the greater is the possibility of being able to see how to deduce the conclusion of the argument to be proved valid.

The second is to try to eliminate statements that occur in the premises, but not in the conclusion. Such elimination can be done only in accordance with the rules of inference. The rules contain many techniques for eliminating statements. Simplification, Commutation, Hypothetical Syllogism are such rules. Distribution is a useful rule for transforming a disjunction into conjunction. All these rules are helpful by various ways in eliminating statements.

The third is to introduce by means of addition a statement that occurs in the conclusion but not in the premises.

So, the most important point is to know the rules well. However, knowing the rules does not mean only memorizing the rules. Memorizing is, no doubt, necessary, but the mastery over the rules is also necessary to understand and help the process.

The fourth is to work backward from the conclusion by looking for some statement or statements from which it can be deduced and then trying to deduce those intermediate statements from the premises.

However, there is no substitute for practice as a method of acquiring facility in constructing formal Proof.

CHECK YOUR PROGRESS

Q 13: For each of the following arguments, state the Rules of Inference by which its conclusion follows from its premise or premises:

A. (A ⊃ ~ B) . (~ C ⊃ D)]

∴A ⊃ ~ B

B. KV (LVM)

∴ [KV (LVM)] V [KV (LVM)]

Q 14: Each of the following is a formal proof of validity for the indicated argument. State the 'Justification' for each line that is not a premise

A. 1. (A .B) ⊃ [A ⊃ (D. E)

2. (A.B). C / DVE

3. A.B

4. A ⊃ (D.E)

5. A

6. D.E

7. D

8. DVE

Q 15: Construct a formal proof of Validity for each of the following arguments.

A A ⊃ B

C ⊃ D

(~ B V D) . (~ A V ~B) / ∴~A V ~C

B. $A \supset B$

$C \supset D$

$A \vee C$ / $\therefore$ $(A . B) \vee (C . D)$

Q 16: Construct a formal proof of validity for each of the following arguments, in each case using the suggested notation.

a. If I study, I make good grades. If I do not study, I enjoy myself. Therefore, either I make good grades or I enjoy myself.(S, G, E)

1.11 LET US SUM UP

- The Method of deduction is a method of establishing the validity of arguments.
- The truth table method is inconvenient for testing the validity of arguments containing a large number of statements.
- So, a more convenient method for testing the validity of arguments is the Method of Deduction or what may be called the method of derivation by substitution.
- Rules of inference and Rules of replacement are the two sets of rules for derivation. By means of these rules, we can know what can be validly inferred from a certain kind of premises.
- The rules of inference work in one direction. These are: Modus Ponens, Modus tollence, Hypothetical Syllogism, Disjunctive syllogism, Constructive Dilemma, Destructive Dilemma, Simplification, Conjunction, Addition.
- Rules of inference are not sufficient for proving the validity of many other arguments. The rules of replacement are required as additional principles in such cases.
- The rules of replacement are: De Morgan's theorem, Commutation, Association, Distribution, Double Negation, Transposition, Material Implication, Material Equivalence, Exportation, Tautology.
- Rules of replacement is indicated by the symbol '≡'.

1.12 FURTHER READINGS

1. Chhanda, Chakrabort (2006). *Logic: Informal, Symbolic and Inductive.* Prentice –Hall of India Private Limited: New Delhi.
2. Copi, Irving M(1979). *Symbolic Logic* (fifth edition). Callier Macmillan International Editions.
3. Copi, Cohen (1995). *Introduction to Logic* (Ninth Edition). New Delhi. Pre New Delhi ntice –Hall of India Private Limited
4. Munshi, R.C. (1991). *Hand book of Logic*. Calcutta. Baikuntha Book House.
5. Singh, S. Shyam Kishore (2000) Modern Logic (Volume one). Lamyana Press

1.13 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q 1: a**)**True b)True c)True

Ans to Q 2: a) Rule of replacement b)Logical equivalences

c)Rule of replacement or the principle of extentionality

Ans to Q 3: The method of deduction is a convenient method for testing the validity of arguments. This method of establishing the validity of argument is to deduce their conclusions from their premises by a sequence of shorter, more elementary arguments each of which is known to be valid.

Ans to Q 4: This deductive procedure relies on the valid argument forms to show as to how the conclusion can be deduced from the given set of premises.

Ans to Q 5: A formal proof of validity for a given argument many be defined "to be a sequence of statements, each of which is either a premiss of that argument or follows from preceding statements by an elementary valid argument, and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved" (I.M . Copi)

Ans to Q 6: An elementary valid argument is any argument that is a substitution instance of an elementary valid argument form.

Ans to Q 7: The Justification in terms of the elementary valid argument form or the logical rule for a new premise specifies the preceding premises from which and the rule by which the premise in question is deduced.

Ans to Q 8: In replacement, a statement can be replaced only by a statement logically equivalent to it. In substituting statements for statements variables in an argument form, any statement variable can have any statement substituted for it. In substitution, every occurrence of a statement variable in an argument form must have the same statement substituted for it.

However, in replacement one or several occurrences of a statement may be replaced by a logically equivalents statement, without other occurrences of that statement having to be replaced also.

Ans to Q 9: Rules of Inference can be applied only to whole lines of proof whereas Rules of Replacement can be applied either to whole lines or to parts of lines. Moreover, in contrast to the Rules of Inference, the Rules of replacement authorize replacement of a statement by its specified logically equivalent statement. Again, Rules of Inference work in one direction while Rules of Replacement work in both directions.

Ans to Q 10: Because, it can be decided mechanically in a finite number of steps whether or not a given sequence of statements constitute a formal proof with reference to the Rules of Inference. Thinking is not required either in thinking about what the statements mean or in using logical intuition to check the validity of deduction. There is a finite mechanical procedure for deciding the legitimacy of deduction even where the justification of a statement is not written beside it.

Ans to Q 11: In constructing a formal proof of validity on the basis of the Rules of Inference, it is necessary to think or 'figure out' where to begin and how to proceed. There is the effective or purely mechanical method of procedure.

Ans to Q 12 :(a) True (b)False (c) True d)True

Ans to Q 13: a) Simplification b) Tautology

Ans to Q 14: A**.** 1. (A. B) ⊃ [A ⊃ (D.E)]

2. (A.B). C / ∴ DvE

3. A. B 2. Simplification

4. A ⊃ (D.E) 1, 3 Modus Ponens

5. A 3 Simplification

6. D.E 4, 5 Modus Ponens

7. D 6 Simplification

8. DVE 7 Addition

Ans to Q 15: A 1. A ⊃B

2. C⊃ D

3. (~B V ~D) . (~A v~ B) / ∴ ~ A V ~C

4. (A ⊃ B) . (C ⊃ D) 1, 2 Conjunction

5. ~ B V ~D 3. Simplification

6. ~ A V ~ C 4, 5 Destructive Dilemma

B. 1. $A \supset B$

2. $C \supset D$

3. A V C / ∴(A. B) V (C.D)

4. $A \supset (A.B)$ 1. Absorption

5. $C \supset (C.D)$ 2. Absorption

6. [$A \supset (A.B)$] . [$C \supset (C. D)$] 6, 3 Constructive Dilemma

Ans to Q 16: 1. $S \supset G$

2. $\sim S \supset E$ / GVE

3. $G \supset \sim S$ 1. Transportation

4. $S \supset \sim S$ 1, 3 Hypothetical Syllogism

5. S v ~ S 4 Material Implication

6. G v E 1, 2, 5 Constructive Dilemma

1. 14 MODEL QUESTIONS

A) Very Short Questions

Q 1: What is method of deduction?

Q 2: What is formal proof of validity?

Q 3: What are the rules of inference?

Q 4: What are the rules of replacement?

B) Short Questions (Answer in about 150 words)

Q 1: What are the elementary valid argument forms? What is their utility in testing arguments?

Q 2: What are the basic points of difference between the rules of inference and the rules of replacement?

C) Long Questions(Answer in about 300-500 words)

Q 1: Explain the strategy of constructing the Formal Proof of validity for arguments.

Q 2: What are the elementary valid argument forms? What is their utility in testing arguments?

Q 3: Construct formal proof of validity for the following:

i) $(A \supset B)$ ii) $(MVN) \supset (O. P)$

	C ⊃ ~ B		~ O /
	∴ A ⊃ ~ C		∴ ~ M

iii)	(M ⊃ N). (O ⊃ P)	iv)	E ⊃ (F ⊃ G)
	~ NV ~ P		∴ F ⊃ (E ⊃ G)
	~ (M. O) ⊃ Q		
	∴ Q		

v.	E ⊃ (F~ G)	vi.	A ⊃ B
	(F VG) V ⊃ ~ H		C ⊃ D
	E		(~ BV ~ D) . (~ AV ~ B)
	∴ H		∴ ~ AV ~ C
vii.	E ⊃ F	viii.	(G ⊃ ~ H) ⊃ I
	G ⊃ F		~ (G.H)
	∴ (E V G) ⊃ F		∴ IV ~ H

ix.	AV (B ⊃ D)	**x.**	E ⊃ (F.~ G)
	~ C ⊃ (D ⊃ E)		(FVG) ⊃ H
	A ⊃ C		E
	~ C		∴ H
	∴ B ⊃ E		

XI. (D . ~ E) ⊃ F

~ (EVF)

∴ ~ D

Q 4: Construct formal proofs of validity for the following arguments using the suggested notation in each case:

i) Either the manager did not notice the change or else he approves of it. He noticed it alright. So he must approve of it. (N, A.).

ii)It is not the case that she either forgot or was not able to finish. Therefore she was able to finish (F,A.)

iii) If either George enrolls or Harry enrolls then Era does not enroll. Either Era enrolls or Harry enrolls. If either Harry enrolls or George does not enroll then Jim enrolls. George enrolls. Therefore either Jim enrolls or Harry does not enroll (G,H, E, J)

iv) If the police do not catch the murderer with a week, then there will be a public outcry. If there is a public outcry, then the chief of the police will resign. The chief of police will not resign. Therefore, the police will of the catch the murderer within a week.