Jacobi's Method:

Consider the system of eq^ns

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \cdots (1)$$

Assume that the diagonal co-efficients a_{11}, a_{22}, and a_{33} are large compared to other co-efficients solving for x_1, x_2 and x_3 respectively.

Now,

$$\left.\begin{aligned} x_1 &= \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3). \\ x_2 &= \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3) \\ x_3 &= \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2) \end{aligned}\right\} \cdots (2)$$

let $x_1^{(0)}$, $x_2^{(0)}$, $x_3^{(0)}$ denote the initial approximations for the values of the unknowns x_1, x_2, x_3 respectively. Substituting these values in the right sides of (2), we get the first iterative values of x_1, x_2, x_3 as follows:

$$x_1^{(1)} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^{(0)} - a_{13}x_3^{(0)}\right)$$

$$x_2^{(1)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{(0)} - a_{23}x_3^{(0)}\right)$$

$$x_3^{(1)} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^{(0)} - a_{32}x_2^{(0)}\right)$$

Substituting the values $x_1^{(1)}$, $x_2^{(1)}$, $x_3^{(1)}$ in the right sides of (2), we get,

$$x_1^{(2)} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^{(1)} - a_{13}x_3^{(1)}\right)$$

$$x_2^{(2)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{(1)} - a_{23}x_3^{(1)}\right)$$

$$x_3^{(1)} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^{(1)} - a_{32}x_2^{(1)}\right)$$

Continue this process till the difference betn two consecutive approximations is as small as we please.

Note: In the absence of any better approximations, the initial approximations for the values of x_1, x_2, x_3 are taken as $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$;

Gauss- Seidel Method:

Consider the system of eqns

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \quad -(1)$$

Suppose in the above system, the co-efficients of the diagonal terms are large in each eqn compared to other co-efficients, solving for x_1, x_2, x_3 respectively.

Now,

$$\left.\begin{aligned} x_1 &= \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3) \\ x_2 &= \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3) \\ x_3 &= \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2) \end{aligned}\right\} \quad ---(2)$$

Let $x_1^{(0)}, x_2^{(0)}, x_3^{(0)}$ denote the initial approximations of x_1, x_2, x_3 respectively. Substituting $x_2^{(0)}$, and $x_3^{(0)}$ in the first eqn of (2), we get,

$$x_1^{(1)} = \frac{1}{a_{11}}(b_1 - a_{12}x_2^{(0)} - a_{13}x_3^{(0)})$$

Then, we substitute $x_1^{(1)}$ for x_1 and $x_3^{(0)}$ for x_3 in the second eqn of (2) which gives

$$x_2^{(1)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{(1)} - a_{23}x_3^{(0)}\right)$$

we substitute $x_1^{(1)}$ for x_1 and $x_2^{(1)}$ for x_2 in the third eqn of (2), which gives

$$x_3^{(1)} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^{(1)} - a_{32}x_2^{(1)}\right)$$

Continue this process till the difference betn two consecutive approximations is as small as we please.