Differential Equations

Unit - 1

Differential equation: An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

Order of a differential equation: The order of the highest order derivative involved in a differential equation is called the order of the differential eqn.

$\frac{dy}{dx} = x + \sin x \longrightarrow$ 1st order

$\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^5 = e^t \longrightarrow$ 4th order

Degree of a differential equation: The degree of a differential equation is the degree of the highest derivative which occurs in it, after the differential equation has been made free from radicals and fractions as far as the derivatives are concerned.

$\frac{dy}{dx} = x + \sin x \longrightarrow$ 1st degree

$\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^5 = e^t \longrightarrow$ 1st degree

$$k\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

$$\Rightarrow \left(k\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2 \times 2}$$

$$\Rightarrow \left(k\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$$

$$\Rightarrow k^2\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 \longrightarrow$$ 2nd degree.

Linear and non-linear diff. eqⁿ:

A diff. eqⁿ is called linear if

(i) every dependent variable and every derivative involved occurs in the first degree only

(ii) no products of dependent variables and/or derivatives occur.

A diff eqⁿ which is not linear is called a non-linear diff. eqⁿ.

eg: Linear diff. eqⁿ:

(a) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

(b) $\frac{dy}{dx} = x + y + 1$

eg: Non-linear:

(a) $\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^2 + y = x^2$

(b) $\frac{dy}{dx} = \frac{x}{y}$

Ordinary diff. eqⁿ: A diff. eqⁿ involving derivatives w.r.t. a single independent variable is called an ordinary diff. eqⁿ.

e.g: (a) $\frac{dy}{dx} = x + \sin x$

(b) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^x$

Partial diff. eqⁿ: A diff. eqⁿ involving partial derivatives w.r.t. more than one independent variable is called a partial diff. eqⁿ.

e.g: (a) $\frac{\partial^2 v}{\partial t^2} = k\left(\frac{\partial^3 v}{\partial x^3}\right)^2$

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$